B.A./B.Sc. 1st Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMHI-CC-I

Time: 3 Hours **Full Marks : 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

   (a) Evaluate the integral $\int \coth^2 x \operatorname{cosech}^2 x\, dx$.

   (b) Find y_n, if $y = x^2 \sin x$.

   (c) Find the envelope of the straight lines $x\cos\theta + y\sin\theta = l\sin\theta\cos\theta$, where l is a fixed constant and θ is the parameter.

   (d) Evaluate: $\lim_{x\to 0} \frac{e^{3x}-3x-1}{1-\cos x}$.

   (e) Evaluate: $\int_0^{\frac{\pi}{2}} \sin^5 x\, dx$.

   (f) Find the values of t for which the parametric curve $x = t^3 - 15t^2 + 24t + 7,\ y = t^2 + 4t + 1$ has

      (i) horizontal tangent line and

      (ii) vertical tangent line.

   (g) Find the total length of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

   (h) Find the area of the surface generated by $y = \sin x$ bounded by the points $x = 0$ and $x = \pi$ when revolved about the x-axis.

   (i) Transform the equation $x^2 - y^2 = 25$ when the axes are rotated through 45°.

   (j) Let $2a$ be the length of the major axis of the ellipse $r = \frac{el}{1+e\cos\theta}$. Show that $a = \frac{el}{1-e^2}$.

   (k) Find the equation of the cylinder whose generators are parallel to the y-axis and which passes through the curve of intersection of the plane $x + y + z = 4$ and the surface $x^2 + y^2 + z^2 = 4$.

(l) If $\frac{x-3}{0} = \frac{y-4}{4} = \frac{z+5}{-5}$ is a generator of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$, then find a, b, c.

(m) State a necessary and sufficient condition for the differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact. Examine whether the differential equation $(2x^3 + 4y)dx + (4x + y - 1)dy = 0$ is exact or not.

(n) Solve : $\frac{dy}{dx} = \frac{6x-2y-7}{3x-y+4}$.

(o) Solve : $x\frac{dy}{dx} + y = -2x^6y^4$.

2. Answer *any four* questions from the following: 5×4=20

(a) (i) Prove that $\cosh 3x = 4\cosh^3 x - 3\cosh x$.

(ii) Write down a condition for a curve to be concave at a point with respect to x-axis. Examine the curve $y = \sin x$ regarding its concavity at $\left(\frac{\pi}{2}, 1\right)$ with respect to x-axis.

2+(1+2)=5

(b) (i) Sketch the graph of $f(x) = \frac{x^2}{\sqrt{x+1}}$.

(ii) The general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, where $h \neq 0$, has the form $a'(x')^2 + b'(y')^2 + 2g'x' + 2f'y' + c' = 0$, when the axes are rotates through an angle θ without any change of origin, then show that $\cot 2\theta = \frac{a-b}{2h}$.

2+3=5

(c) (i) Find the vertex, focus and the length of the latus rectum of the Principal sections of the paraboloid $4x^2 - 9y^2 = 36z$.

(ii) Show that the plane $x + y - z = 0$ cuts the conicoid

$4x^2 + 2y^2 + z^2 + 3yz + zx - 1 = 0$ in a circle. What is the radius of the circle? 2+3=5

(d) (i) Evaluate : $\int \text{cosec}^5 x\, dx$

(ii) Find the length of the curve $y = \cosh\frac{x}{a}$ from $x = 0$ to $x = a,\ a > 0$. 2+3=5

(e) (i) Find the greatest and the least distances from the point (2, –1, 1) to the sphere $x^2 + y^2 + z^2 - 8x + 4y - 6z + 4 = 0$.

(ii) Find the equations to the generating lines of the hyperboloid $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$ which pass through the point (2, 3, –4).

2+3=5

(f) (i) Solve : $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$

(ii) Solve : $y^2 \log_e y = xyp + p^2,\ p \equiv \frac{dy}{dx}$ 3+2=5

3. Answer *any two* questions from the following: 10×2=20

(a) (i) Find the asymptotes of the following curve:

$$3x^3 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0.$$

(ii) Find the set of values of x for which the following curve is concave upwards:

$$y = 3x^5 - 40x^3 + 3x - 20$$

(iii) Let the income tax function $T(x)$ be defined as follows:

$$\begin{aligned} T(x) &= 0, \text{ if } 0 \leq x < 2{\cdot}5 \\ &= \frac{1}{20}(x - 2{\cdot}5), \text{ if } 2{\cdot}5 \leq x < 5 \\ &= \frac{1}{8} + \frac{1}{5}(x-5), \text{ if } 5 \leq x < 10 \\ &= \frac{9}{8} + \frac{3}{10}(x - 10), if\ x \geq 10\ . \end{aligned}$$

Find the points of non-differentiability, if any, of the function $T(x)$ and interpret it.

4+3+3=10

(b) (i) Obtain a reduction formula for $I_{m,n} = \int \sin^m x \cos^n x dx$ where either m or n or both are negative integers with $m, n \neq -1$.

(ii) Find the surface area of the anchor ring formed by the revolution of a circle of radius a about a line in its plane at a distance b $(b > a)$ from its centre.

(iii) An astroid $x = a\cos^3\theta,\ y = a\sin^3\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$ revolves about the x-axis. Find the surface area of the solid generated. 4+3+3=10

(c) (i) Reduce the following equation to its canonical form and determine its nature:

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$

(ii) Find the equation of the right circular cylinder whose guiding curve is the circle through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1).

(iii) Determine the angle between the lines of intersection of the plane $x - 3y + z = 0$ and a quadric cone $x^2 - 5y^2 + z^2 = 0$. 4+3+3=10

(d) (i) Reduce the equation $(px^2 + y^2)(px + y) = (p + 1)^2, p \equiv \frac{dy}{dx}$, to Clairaut's form by using the substitution $u = xy$ and $v = x + y$ and hence find its complete primitive.

(ii) Solve : $(e^x \sin y + e^{-y})dx + (e^x \cos y - xe^{-y})dy = 0$

(iii) Solve : $(1 + y + x^2y)dx + (x + x^3)dy = 0$ and find the particular solution if $y = 1$ when $x = 1$.

4+3+3=10