B.A./B.Sc. 2nd Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMH2CC04

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Solve : $(x+y)^2 \frac{dy}{dx} = a^2$, '$a$' being a constant.

(b) Find the Wronskian of a pair of linearly independent solutions of $\frac{d^2y}{dx^2} + 9y = 0$.

(c) Show that the given problem has infinite number of solutions: $\frac{dy}{dx} = \frac{y-1}{x}, y(0) = 1$.

(d) Distinguish between initial value problem and boundary value problem.

(e) Find the regular singular points of the differential equation:
$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$.

(f) if $\vec{\alpha}\times\vec{\beta} + \vec{\beta}\times\vec{\gamma} + \vec{\gamma}\times\vec{\alpha} = 0$, then show that $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are coplanar.

(g) State the existence theorem on the solution of the ordinary differential equation $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$.

(h) Find the particular integral of the differential equation: $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} = 2$.

(i) Show that the substitution $z = \sin x$ transform the differential equation:
$\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$ to $\frac{d^2y}{dz^2} + y = 0$.

(j) If $\vec{f} = \vec{a}\cos t + \vec{b}\sin t$, then prove that $\vec{f}\cdot\left(\frac{d\vec{f}}{dt}\times\frac{d^2\vec{f}}{dt^2}\right) = 0$, where $\vec{a}$ and $\vec{b}$ are constant vectors.

(k) Evaluate: $\int_0^1 \left(\vec{f}\cdot\frac{d\vec{f}}{dt}\right)dt$, where $\vec{f}(0) = \vec{i}+\vec{j}+\vec{k}$ & $\vec{f}(1) = 2\vec{i}+3\vec{j}+4\vec{k}$.

(l) Consider the system of differential equations
$$\frac{dx}{dt} = 2x - 7y.$$
$$\frac{dy}{dt} = 3x - 8y$$
Discuss the nature of the critical point (0, 0) of the system.

(m) Find a non-trivial solution of $\frac{d^2y}{dx^2} - 4y = 0,\ y(0) = 0,\ y(\pi) = 0$.

(n) Find the interval of x, where the solution of $\frac{dy}{dx} = 1 + y^2, y(0) = 1$ exists.

(o) Find $\frac{d}{dt}\left(\vec{r}^2 + \frac{1}{\vec{r}^2}\right)$.

2. Answer *any four* questions from the following: 5×4=20

(a) Apply the method of variation of parameters to solve the differential equation:
$\frac{d^2y}{dx^2} + y = \sec x \tan x$. 5

(b) Find the general solution of the following differential equation:
$\frac{d^2y}{dx^2} + 4y = 12x^2 - 16\cos 2x$ 5

(c) Solve the differential equation $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = (x-2)e^x$, by the method of undetermined coefficients. 5

(d) Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6$, subject to the conditions $y(0) = 3, y'(0) = 1$. Does the problem have unique solution? Justify your answer. 3+1+1=5

(e) Show that $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$, unless $\vec{b}$ is parallel to $(\vec{a} \times \vec{c})$ or $\vec{a}$ and $\vec{c}$ are collinear. 3+2=5

(f) Prove that the continuity of $f(x,y)$ in the neighbourhood of the origin is not enough to guarantee the uniqueness of the solution of the initial value problem:
$\frac{dy}{dx} = f(x,y) = \sqrt{|y|},\ y(0) = 0.$ 5

3. Answer *any two* questions from the following: 10×2=20

(a) (i) Show that a linear homogeneous differential equation of order n cannot have more than n linearly independent solutions.

(ii) If f is a differentiable function over $\mathbb{R}$ and satisfies the equation $f(x+y) = f(x)f(y)$, then show that either $f(x) = 0$ or $f(x) = e^{\alpha x}$, where α is a constant. 5+5=10

(b) (i) Solve the following system of equations:
$$\frac{dx}{dt} = x + 3y$$
$$\frac{dy}{dt} = 3x + y$$

(ii) Solve the equation $2x^2y\frac{d^2y}{dx^2} + 4y^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$ after making it homogeneous, by substituting $y = z^2$. 5+(3+2)=10

(c) (i) Find the power series solution of the differential equation:
$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$ about $x = 0$.

(ii) For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ show that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. 5+5=10

(d) (i) Evaluate $\int_2^3 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2}\right) dt$ where $\vec{r} = t^3\,\hat{\imath} + 2t^2\,\hat{\jmath} + 3t\,\hat{k}$.

(ii) If $\vec{r}(t) = \begin{cases} 2\vec{\imath} - \vec{\jmath} + 2\vec{k} & \text{when } t = 2 \\ 4\vec{\imath} - 2\vec{\jmath} + 3\vec{k} & \text{when } t = 3 \end{cases}$, find $\int_2^3 \left(\vec{r} \cdot \frac{d\vec{r}}{dt}\right) dt$.

(iii) Show that the equation $\frac{dy}{dx} = 3xy^{1/3}, y(0) = 0$ does not have unique solution. 2+3+5=10
