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B.SC. 4th Semester (Honours) Examination, 2019 (CBCS)

Subject : Physics

Paper : CC-VIII

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* of the following questions: 2×5=10

(a) Find the smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$.

(b) Simplify the expression $Z = i^{-2i}$.

(c) Examine whether the function $f(z) = \sin z$ is analytic or not.

(d) What do you mean by branch points?

(e) Find the Fourier transform of the function $f(x) = 1$ for $|x| < 1$
$= 0$ for $|x| > 1$

(f) State Fourier convolution theorem.

(g) Find the Laplace transform of the function $F(t) = \cos\left(t - \frac{2\pi}{3}\right)$ for $t > \frac{2\pi}{3}$
$= 0$ for $t < \frac{2\pi}{3}$

(h) If $f(s) \equiv L\{F(t)\}$ then show that $L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$, where L indicates Laplace transform.

2. Answer *any two* of the following questions: 5×2=10

(a) State Taylor's theorem. Find the Taylor series expansion of the complex variable function $f(z) = \frac{z+1}{(z-3)(z-4)}$ about $z = 2$. Find the region of convergence. 1½+3+½=5

(b) Using the residue theorem, show that $\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta} = \frac{2\pi}{\sqrt{a^2+b^2}}$ if $a > |b|$. 5

(c) Define Dirac delta function $\delta(t)$. Find the Fourier transform of the Dirac delta function $\delta(t)$ given below and then by inverting the transform obtain an integral expression for the Dirac delta function:

$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-iwt}\, dw.$ 1+2+2=5

(d) Using Laplace transform, solve $\frac{\partial U(x_1 t)}{\partial t} = \frac{\partial^2 U(x_1 t)}{\partial x^2}$, with the conditions $U(x, 0) = 3\sin 2\pi x$, $U(0, t) = 0, U(1, t) = 0$, where $0 < x < 1, t > 0$. 5

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10×2=20

3. Answer *any two* of the following questions:

(a) (i) Prove that, $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$.

(ii) Let (r,θ) denote the point $r(\cos\theta+\mathrm{i}\sin\theta)$ in the Argand plane. If $a\equiv(1,\alpha)$, $b\equiv(1,\ \beta)$, $c\equiv(1,\gamma)$ and $a+b+c=0$, show that $a^{-1}+b^{-1}+c^{-1}=0$.

(iii) Write the Cauchy-Riemann equations. In which domain(s) of the complex plane is $f(z)=|x|-i\,|y|$ an analytic function?

3+4+(1+2)=10

(b) (i) Write Cauchy's integral formula. Evaluate $\oint_C \frac{dz}{z-a}$, where C is any simple closed curve C and $Z=a$ is:

(I) outside C

(II) inside C.

(ii) Locate and classify the singularities of the following functions of complex variable z:

(A) $\frac{e^z-1}{z}$ (B) $e^{\frac{1}{z-2}}$

(iii) Evaluate $\frac{1}{2\pi i}\oint_C \frac{e^{zt}}{z^2(z^2+2z+2)}dz$, around the circle with the equation $|z|=3$.

(1+1+2)+(1+1)+4=10

(c) (i) Find the Fourier transform of the function $f(x)=1-x^2$ if $|x|\leq 1$.
$=0$ if $|x|>1$

(ii) Using Fourier transform, solve the one dimensional wave equation:
$\frac{\partial^2 y(x,t)}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$, with the initial condition $y(x,0)=f(x)$. Here v is the velocity of the wave.

5+5=10

(d) (i) Find the inverse Laplace transform of $f(s)=\frac{a^2}{s(s+a)^2}$.

(ii) Using Laplace transform method, solve the Damped Harmonic oscillator equation:
$mX''(t)+bX'(t)+kX(t)=0$, with the conditions $X(0)=X_0, X'(0)=0$.

4+6=10

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