B.Sc. Semester V (Honours) Examination, 2020 (CBCS)

Subject: Physics

Paper: CC-XI

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as per as practicable.

Answers any ***eight*** of the following questions (All questions carry equal marks): 5×8=40

1. a) Why should a wave function be finite and continuous everywhere?

   b) A free-particle wave function can be written as

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} e^{-i(kx-\omega t)}$$

   Find the energy of the particle using this wave function and the time-independent Schrödinger equation.

   c) When can a dynamical variable be called an "observable" in quantum mechanics?

2. a) What are stationary states? Why is an energy eigen state called a stationary state?

   b) The wave function of a particle in a stationary state with an energy E_0 at time $t = 0$ is ψ (x). After how much minimum time will the wave function be again ψ (x)?

   c) Find the eigen values of the linear momentum operator.

3. a) Show that

$$\left(x_i p_j - p_j x_i\right)\psi = i\hbar\delta_{ij}\psi \text{ (Symbols have their usual meaning).}$$

   b) Prove that the momentum of a free particle commutes with the Hamiltonian operator.

   c) Prove that operators having a common set of eigen functions commute.

4. a) Consider a linear harmonic oscillator for which the total energy is given by

$$E = \frac{p_x^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

   where symbols have their usual meaning. The particle is assumed to be confined to a region ~ a. Using Heisenberg's uncertainty principle, obtain the ground state energy of the oscillator.

5. A particle of mass m is confined in a one-dimensional box of rigid walls so that it is in the following potential:

$$V = 0 \quad \text{for } -a < x < a$$

$$V \to \infty \quad \text{for } x < -a \text{ and } x > a$$

   The wave function of this particle is found to be $\psi = A\left(\cos\frac{\pi x}{2a} + \sin\frac{3\pi x}{a} + \frac{1}{4}\cos\frac{3\pi x}{2a}\right)$, inside the wall, and ψ = 0 outside the wall. Calculate A such that the wave function is normalized.

6. a) A particle at $t = 0$ is described by a one-dimensional square wave packet

$$\psi_0(x) = \begin{cases} \frac{1}{\sqrt{2a}} e^{ik_0 x} & |x| \le a \\ 0 & |x| > a \end{cases}$$

Find φ (**k**), the probability amplitude for measuring a wave vector **k**.

b) Find position probability density and position probability current density for the following normalized Gaussian wave packet

$$\psi(x) = \frac{1}{\sqrt{\sigma\sqrt{\pi}}} e^{-\frac{(x-x_0)^2}{2\sigma^2}} e^{i\frac{p_0 x}{\hbar}}$$

7. In a Stern-Gerlach experiment with silver, the following data were obtained:
Length of the magnetic field = 0.04 m
Distance of screen from mid-point of magnetic field = 0.12 m
Initial speed of the silver atoms = 500 m s^{-1}.
Rate of variation of flux density = 1.5 T mm^{-1}.
Maximum separation between the two traces = 3 mm
Mass of silver atom = 1.7911×10^{-25} Kg.

Compute the value of the magnetic moment of the silver atom in the direction of the field.

8. a) Determine the state which can be formed from a 2-electron configuration in the L-S coupling scheme given that $l_1 = 1$, $l_2 = 3$.

b) What are symmetric and antisymmetric wave functions for a system of two identical particles?

9. Explain the "spin-orbit coupling" of atomic electron and consequent doubling of spectral lines with the necessary expressions.

10. Calculate the expectation value of the kinetic energy of the electron in the 1S state of the hydrogen atom.

Given $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$

and $\psi_{100}(r,\theta,\varphi) = \frac{1}{\sqrt{\pi}}\frac{1}{a_0^{3/2}} e^{-\frac{r}{a_0}}$

where the notations have usual meaning.