B.A/B.Sc 6th Semester (Honours) Special Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH6DSE41
(Bio Mathematics)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: **6×5=30**

(a) Write a short note on Allee effect.

(b) Which type of model you can assume for the following set of arbitrary population (in millions) data. Justify your answer.

Year	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
Population	1700	1800	2100	2200	2400	2800	3700	4400	5200	6100	6850

(c) Discuss Holling type growth.

(d) Explain environmental carrying capacity in logistic growth.

(e) Find the interior equilibrium point of the following model

$$\frac{dx}{dt} = rx(1-x/k) - \frac{axy}{b+x}$$
$$\frac{dy}{dt} = \frac{cxy}{b+x} - dy$$

where r, k, a, b, c and d are all positive constants.

(f) Discuss Spruce Budworm model.

(g) Find the general solution to the following homogeneous difference equation

$$x_{t+3} + 5x_{t+2} - x_{t+1} - 5x_t = 0.$$

(h) What do you mean by reaction diffusion equation? Explain with an example. 3+2

2. Answer any three questions: **3×10=30**

(a) State and prove the Routh-Hurwitz criteria for a second order polynomial equation. Hence discuss the nature of the roots of the characteristic equation of the following differential equation

$$\frac{d^2x}{dt^2}+\frac{dx}{dt}+3x=0$$ 2+4+4

(b) Analyse the phase plane of the following system

$$\frac{dx}{dt}=ax+by$$
$$\frac{dy}{dt}=cx+dy$$

where a,b,c,d are all positive constants.

(c) Show that every solution of the following system with positive initial conditions is periodic.

$$\frac{dx}{dt}=x(1-y)$$
$$\frac{dy}{dt}=\alpha y(x-1)$$

where α is a positive constant.

(d) (i) Solve the following unidirectional nonlinear wave motion with initial value using the method of characteristics:

$$u_t+uu_x=0,\ t\in(0,\infty),\ x\in(-\infty,\infty)$$

with $u(0,x)=\phi(x),\ x\in(-\infty,\infty)$

(ii) Reduce a two species diffusion model into a linearized system around any specially uniform steady state. 5+5

(e) (i) Construct cobweb maps for $N_{t+1}=\dfrac{(1+r)N_t}{1+rN_t}$ where r is a positive constant. Discuss global behavior of the solution.

(ii) Find the fixed point of the following system

$$N_{t+1}=N_t+rN_t\left(1-\frac{N_t}{k}\right)-eN_tP_t$$
$$P_{t+1}=bN_tP_t+(1-d)P_t$$

where r,k,e,b,d are all positive constants. 5+5

B.A/B.Sc 6th Semester (Honours) Special Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMH6DSE42

(Differential Geometry)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5=30

(a) Prove that any tangent developable surface is isometric to a plane.

(b) If γ is any space curve, then obtain a formula to determine its curvature.

(c) Compute the curvature of a right circular helix.

(d) What are the geodesics on a sphere and right circular cylinder?

(e) Deduce a relation of the three fundamental forms of a surface.

(f) Determine the principal curvatures of the right circular cylinder.

(g) If $\gamma(t) = \sigma\big(u(t), v(t)\big)$ is a unit speed curve on a surface patch σ, then deduce a formula to determine its normal curvature.

(h) Prove that surface of a sphere is a smooth surface.

2. Answer any three questions: 3×10=30

(a) State and prove fundamental theorem of a space curve.

(b) Obtain a characterization of a general helix.

(c) Deduce Serret-Frenet Formulae for a space curve.

(d) Deduce the Gaussian curvature of a unit sphere.

(e) State and prove Meusnier's theorem.

B.A/B.Sc 6th Semester (Honours) Special Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH6DSE43
(Mechanics-II)

Time: 3 Hours — Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. **Answer any six questions:** **6 × 5= 30**

(a) Obtain the relation between volume and temperature in an adiabatic change of gases.

(b) A fluid is in equilibrium under a given system of forces. If $\rho_1(x,y,z)$ and $\rho_2(x,y,z)$ be two possible values of the density at any point, then prove that there exists a function *f* such that $\rho_1/\rho_2 = f(p_2)$ where p_2 is the pressure corresponding to the density ρ_2.

(c) In a conservative field of force, prove that the surfaces of equi-pressure, equi-density and equi-potential energy coincide.

(d) A solid cylinder of radius a and length h is floating with its axis vertical. Show that the equilibrium will be stable if

$$\frac{a^2}{l} > 2(h-l)$$ where l is the length of the axis immersed?

(e) A particle is constrained to move on the plane curve $xy = c$ (where c is a constant) under the gravity, y-axis vertical. Obtain the Lagrange's equation of motion.

(f) A cylindrical tumbler, half filled with a liquid of density ρ, is filled up with a liquid of density ρ', which does not mix with the former one. Show that the thrust on the base of the tumbler is to the whole pressure on its curved surface as $2r(\rho+\rho')$ to $h(\rho+3\rho')$ where h is the base of the tumbler and r is the radius of its base.

(g) A gas at uniform temperature is acted upon by the force

$$X = \frac{-y}{x^2+y^2}\ ,\ Y = \frac{x}{x^2+y^2}$$

Find its density.

(h) Obtain differential equation for curves of equi-pressure and equi-density.

2. Answer any three questions: 3 x 10 = 30

(a) Define stress quadric at a point of a continuous medium. State and prove any one property of stress quadric.

(b) The Lagrangian L for the motion of a particle of unit mass is

$$L=\frac{1}{2}\,(\dot{x}^2+\dot{y}^2+\dot{z}^2)-V+A\dot{x}+B\dot{y}+C\dot{z}$$

where each of V, A, B, C is a given function of (x,y,z). Show that the equations of motion can be written in the form

$$\ddot{\vec{r}}=-\vec{\nabla}V+\dot{\vec{r}}\times curl\,\vec{s}$$

where $\vec{r}=(x,y,z)$, $\vec{s}=(A,B,C)$

(c) Prove that if the force per unit mass at (x,y,z) parallel to the axes are $y(a-z), x(a-z), xy$, the surfaces of equi-pressure are hyperbolic paraboloid and the curves of equal pressure and density are rectangular hyperbolas.

(d) A mass M of gas at uniform temperature is diffused through all space and at each point (x,y,z), the components of force per unit mass are $-x,-y,-z$. The pressure and density at origin are p_0 and ρ_0 respectively. Prove that

$$\rho_0=\frac{8\pi^3 p_0^3}{M^2}$$

(e) For a scleronomic dynamical system, show that the kinetic energy is a homogeneous quadratic function of generalized velocities.