B.Sc. Semester III (Honours) Examination, 2021 (CBCS)

Subject: Physics

Paper: CC-V

Time: 2 Hours **Full Marks: 40**

The questions are of equal value. Candidates are required to give their answers in their own words as far as practicable.

Answer any **eight** questions: 5x8=40

1. Prove that
   (i) $\frac{d}{dx}[\text{erfc}(ax)] = -\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$
   (ii) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

2. Prove that
   (i) $\Gamma(2m) = 2^{2m-1}\pi^{-\frac{1}{2}}\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)$.
   (ii) $\left(m+\frac{1}{2}\right)! = \frac{\pi^{\frac{1}{2}}(2m+1)!!}{2^{m+1}}$, where $(2m+1)!! = 1.3.5 \ldots\ldots (2m-1)(2m+1)$

3. Expand $f(x) = \begin{matrix} 1+\frac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & 0 \le x \le \pi \end{matrix}$ in Fourier series and hence find the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..$

4. (i) Write down the expression for Fourier integral representation of (x) .
   (ii) Express $f(x) = \begin{matrix} 1 & 0 \le x \le \pi \\ 0 & x > \pi \end{matrix}$ as Fourier sine integral and hence find the value of
   $\int_0^\infty \frac{1-\cos\pi\omega}{\omega}\sin\omega x\, d\omega$

5. Solve the equation in series; $\frac{d^2y}{dx^2} - y = 0$.

6. Write down the generating function for Bessel's differential equation. Why is it so called? Prove that $J_n(x)J'_{-n}(x) - J'_n(x)J_{-n}(x) = \frac{2\sin n\pi}{\pi x}$

7. What is Rodrigue formula for Legendre polynomial? Prove the same. Hence draw the graph of $P_0(x)\ and\ P_1(x)$

8. Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ in the interval $0 \le x \le \pi$ with boundary condition i)$u(0,y)=0,\ ii)u(\pi,y)=0;\ iii)u(x,0)=1;\ iv)u(x,y)\to 0\ as\ y\to\infty$.

9. The diameter of a semicircular plate of radius a is kept at 0 deg and the temperature at a semicircular boundary is $k\theta(\pi-\theta)$. Show that the steady state temperature in the plate is given by $u(r,\theta)=\frac{8k}{\pi}\sum_1^\infty \frac{1}{(2n-1)^3}(\frac{r}{a})^{2n-1}\sin(2n-1)\,\theta$

10. a) What is prportional error and percentage error for a continuous function $f(x,y)$, when $\delta x\ and\ \delta y$ are the increments in $x\ and\ y$ respectively.

    b) If the time period of a simple pendulum with length l is $T=2\pi\sqrt{\frac{l}{g}}$, find the maximum percentage error in T when the possible error is 1% in l and 2.5% in g.