B.A/B.Sc 5th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH5CC12
(Mechanics-I)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) (i) For a system of coplanar forces, explain the concept of astatic centre geometrically. [2]

(ii) For a system of coplanar forces acting on a rigid body, find the condition(s) of astatic equilibrium. [3]

(b) A force P acts along the axis of x and another force nP acts along a generator of the cylinder $x^2 + y^2 = a^2$, show that the central axis lies on the cylinder [5]

$$n^2(nx-z)^2+(1+n^2)^2y^2=n^4a^2 .$$

(c) A heavy uniform elliptical wire of semi axes a**,** b is hung over a small rough peg. Show that, if the wire can be in equilibrium with any point in contact with the peg, the coefficient of friction cannot be less than $\dfrac{a^2-b^2}{2ab}$. [5]

(d) Show that the differential equation of the path of a particle in a plane curve under a central attractive force F is $u+\dfrac{d^2u}{d\theta^2}=\dfrac{F}{h^2u^2}$. [3+2]

Also prove that $v^2=h^2\left[u^2+\left(\dfrac{du}{d\theta}\right)^2\right]$.

(e) Discuss the effects of a periodic disturbing force on a harmonic oscillator. [5]

(f) Discuss the motion of a heavy particle on a rough inverted cycloid. [5]

(g) A wire is in the form of a semi-circle of radius a. Show that at an end of its diameter, the principal axes in its plane are inclined to the diameter at angles [5]

$$\frac{1}{2}\tan^{-1}\frac{4}{\pi} \quad \text{and} \quad \left(\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\frac{4}{\pi}\right).$$

(h) Show that $MK^2\dfrac{d^2\theta}{dt^2}$ represents the moment about centre of inertia of all external forces acting on the system. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) A regular hexagon is composed of six equal heavy rods freely jointed together and two opposite angles are connected by a string, which is horizontal, one rod being in contact with a horizontal plane; at the middle point of the opposite rod a weight W_1 is placed; if W be the weight of each rod , show that the tension of the string is $\dfrac{3W+W_1}{\sqrt{3}}$. [6]

(ii) M_1 , M_2 , M_3 are the moments of a system of forces acting in the *xy*-plane about three non-collinear points $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ respectively. If the resultant of the system is a single force at the origin, show that [4]

$$M_1(x_2y_3-x_3y_2)+M_2(x_3y_1-x_1y_3)+M_3(x_1y_2-x_2y_1)=0\ .$$

(b) (i) Find the centre of gravity of a plate in the form of a quadrant AOB of an ellipse, the thickness at any point of the plate varying as the product of the distances of the point from OA and OB. [5]

(ii) Define a common catenary. Deduce the cartesian equation of a common catenary. [1+4]

(c) (i) A particle is projected vertically upwards with a velocity V from the earth's surface. If h and H are the greatest heights attained by the particle moving under uniform and variable acceleration respectively, show that $\dfrac{1}{h}-\dfrac{1}{H}=\dfrac{1}{R}$, [5]

where R is the radius of the earth.

(ii) Find the escape velocity of a particle moving under a central force. [5]

(d) (i) A particle falls vertically from rest in a medium whose resistance varies as the square of the velocity; investigate the motion of the particle. [5]

(ii) A particle falls down a cycloid under its own weight starting from the cusp. Show that when it arrives at the vertex the pressure on the curve is twice the weight of the particle. [5]

(e) (i) Deduce the kinetic energy of a rigid body rotating about a fixed point, in terms of its angular velocity and its principal moments of inertia. [5]

(ii) A uniform rod is held at an inclination λ to the horizon with one end in contact with a horizontal table whose coefficient of friction is μ. If it then be released, show that it will commence to slide if [5]

$$\mu<\frac{3\sin\lambda\cos\lambda}{1+3\sin^2\lambda}\ .$$