B.A/B.Sc 5th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH5DSE11
(Linear Programming)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Solve the following linear programming problem : [5]

Maximize $Z = 2x_1 - 3x_2$
subject to the constraints

$$-x_1 + x_2 \geq -2$$
$$5x_1 + 4x_2 \leq 46$$
$$7x_1 + 2x_2 \geq 32$$
$$x_1, x_2 \geq 0.$$

(b) A Company produces two types of sauces: A and B. These sauces are both made by blending two ingredients X and Y. A certain level of flexibility is permitted in the formulae of these products. Indeed the restrictions are that i) B must contain not more than 75 percent of X and ii) A must contain not less than 25 percent of X and not less than 50 percent of Y. Upto 400 kg of X and 300 kg of Y could be purchased. The company can sell as much as these sauces as it produces at a price Rs. 18 for A and Rs 17 for B per kg. X and Y cost Rs.1.60 and Rs.2.05 per kg respectively. Formulate a linear programming problem to maximize its profit. [5]

(c) (i) Define convex set with an example. [2]

(ii) Prove that the set of all feasible solutions of $Ax = b$, $x \geq 0$ is a closed convex set. [3]

(d) Use the Dual Simplex method to solve the problem given below: [5]

Minimize $Z = x_1 + 2x_2$
Subject to $2x_1 + x_2 \geq 4$
$x_1 + 7x_2 \geq 7$
$x_1, x_2 \geq 0.$

(e) Prove that a necessary and sufficient condition for the existence of feasible solution of a transportation problem is $\sum a_i = \sum b_j$, $(i = 1,2, \ldots, m; j = 1,2, \ldots, n)$. [5]

(f) Find the optimal solution of the following Transportation Problem : [5]

	D_1	D_2	D_3	D_4	a_i
O_1	23	27	16	18	30
O_2	12	17	20	51	40
O_3	22	28	12	32	53
b_j	22	35	25	41	123

(g) Solve the following Travelling Salesman problem : [5]

To

From

	A	B	C	D	E
A	∞	6	12	6	4
B	6	∞	10	5	4
C	8	7	∞	11	3
D	5	4	11	∞	5
E	5	2	7	8	∞

(h) Using dominance property reduce the following payoff matrix to 2×2 matrix and hence solve the problem : [5]

Player B

Player A

	B_1	B_2	B_3	B_4
A_1	3	2	4	0
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

2. Answer any three questions: 10×3 = 30

(a) (i) Prove that every extreme point of the convex set of all feasible solutions of the system $Ax = b, x \geq 0$ corresponds to a basic feasible solution. [5]

(ii) Find all the basic feasible solutions of the following system of equations [5]

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

(b) (i) Solve the following LPP by Big-M method : [6]

Maximize Z = $6x_1 + 4x_2$

subject to $2x_1 + 3x_2 \leq 30$

$$3x_1 + 2x_2 \leq 24$$
$$x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0.$$

(ii) Solve the following LPP by two phase method : [4]

$$\text{Maximize } Z = 5x_1 + 3x_2$$
$$\text{subject to} \quad 2x_1 + x_2 \leq 1$$
$$3x_1 + 4x_2 \geq 12$$
$$x_1, x_2 \geq 0.$$

(c) (i) If the *i*-th variable in the primal problem is unrestricted in sign then show that the *i*-th constraint of the corresponding dual problem is an equation. [3]

(ii) Using duality, solve the following problem: [7]

$$\text{Maximize } Z = 5x_1 - 2x_2 + 3x_3$$
$$\text{subject to} \quad 2x_1 - 2x_2 + x_3 \geq 2$$
$$3x_1 - 4x_2 \leq 3$$
$$x_2 + 3x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0.$$

(d) (i) Solve the following assignment problem : [6]

	M_1	M_2	M_3	M_4
J_1	2	3	4	5
J_2	4	5	6	7
J_3	7	8	9	8
J_4	3	5	8	4

(ii) Show that a subset X of column vectors of coefficient matrix of a transportation problem is linearly dependent if their corresponding cells in the transportation tableau contains a loop. [4]

(e) (i) By the graphical method solve the game whose payoff matrix is given by: [5]

Player B

Player A		B_1	B_2	B_3	B_4
	A_1	6	5	2	3
	A_2	1	2	6	3

(ii) Solve the following game: [5]

Player B

Player A		B_1	B_2	B_3	B_4
	A_1	4	2	3	2
	A_2	-2	4	6	4
	A_3	2	1	3	5

B.A/B.Sc 5th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH5DSE12
(Number Theory)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) Solve the congruence $x^2 \equiv 91(mod\ 3^3)$. [5]

(b) Define Legendre Symbol. Prove that $\left(\frac{ab}{p}\right) = \left(\frac{a}{p}\right)\left(\frac{b}{p}\right)$ where p is an odd prime a, b are any integers coprime to p. [5]

(c) Prove that $\varphi(n) = \frac{n}{2}$ if and only if $n = 2^k$ for some integer $k \geq 1$, where φ is the Euler's phi function. [5]

(d) Find all primitive roots modulo 11. [5]

(e) Show that, if $gcd(a, n) = gcd(b, n) = gcd\left(ord_n^a, ord_n^b\right) = 1$ then $ord_n^{ab} = ord_n^a . ord_n^b$. [5]

(f) Prove that, if a is prime to b then $a^2 + b^2$ is prime to a^2b^2. [5]

(g) Find the values of the Legendre Symbols: $\left(\frac{180}{59}\right), \left(\frac{1236}{4567}\right)$. [5]

(h) Find the general solution in integer of the equation $12x + 7y = 8$. [5]

2. Answer any three questions: $10\times3 = 30$

(a) (i) Prove that there are infinitely many primes of the form $4k + 1, k$ being an integer. [5]

(ii) Prove that $12|xyz$ for any primitive Pythagorean triple x, y, z. [5]

(b) (i) If p is prime, prove that $\sqrt{p}$ is not a rational number. [5]

(ii) Prove that the total number of positive divisors of a positive integer n is odd if and only if n is a perfect square. [5]

(c) (i) Show that Goldbach conjecture (G) implies that every even integer > 5 is sum of three primes. [5]

(ii) Find a four digit numbers which satisfy the following properties: (i) perfect square (ii) first two digits are equal (iii) last two digits are equal. [5]

(d) (i) Find the least positive integer which leaves remainder 2, 3 and 4 when divided by 3, 5 and 11 respectively. [7]

(ii) Prove that the functions τ and σ are both multiplicative. [3]

(e) (i) Prove that there are no primitive roots belonging to 2^n for all $n \geq 3$. [5]

(ii) Find the remainder when $1^3 + 2^3 + \cdots + 99^3$ is divided by 3. [5]

B.A/B.Sc 5th Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH5DSE13
(Point Set Topology)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Prove that a function $f:(X,\tau)\to(Y,\tau')$ is continuous if and only if $\overline{f^{-1}(B)}\subset f^{-1}(\overline{B})$ for every $B\subset Y$; (X,τ) and (Y,τ') being any two topological spaces. [5]

(b) Prove that the union of a family of connected sets, no two of which are separated is a connected set. [5]

(c) Define a locally connected space. Give an example of a connected space which is not locally connected. [1+4]

(d) Prove that the image of a locally connected space under a mapping which is both open and continuous is locally connected. Is the continuous image of a locally connected space is locally connected? Support your answer. [3+2]

(e) Prove that every complete metric space is of second category. [5]

(f) Prove that a compact subset in a metric space is closed and bounded. Is the converse true? Support your answer. [2+2+1]

(g) If u is an infinite cardinal number, prove that $uu=u$. [5]

(h) If u, v and w are cardinal numbers, prove that $u^v u^w = u^{v+w}$. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) If u, v, w are cardinal numbers and $u\le v$, prove that $uw\le vw$. [3]

(ii) For any cardinal number u, prove that $u<2^u$. [4]

(iii) Prove that $2^a = c$, where $a = card\,\mathbb{N}$ and $c = card\,\mathbb{R}$. [3]

(b) (i) Define an ordinal number. If α,β,γ are order types with $\alpha<\beta$ and $\beta<\gamma$, prove that $\alpha<\gamma$. [1+4]

(ii) For any two ordinal numbers α and β, prove that exactly one of the following holds: $\alpha<\beta, \alpha=\beta, \beta<\alpha$. [5]

(c) (i) Define a Kuratowski closure operator and explain the topology derived from it. [1+4]

(ii) Let A be a subset of a topological space (X,τ) and $x_0\in X$. Let $\{x_n\}$ be a [2+3] sequence in A such that $\{x_n\}$ converges to x_0. Prove that $x_0 \in \overline{A}$. Is the converse true? Justify your answer.

(d) (i) Prove that an infinite space with cofinite topology is compact. [3]

(ii) Prove that a closed subspace of a locally compact space is locally compact. [3]

(iii) Give an example with justification to show that the continuous image of a locally compact space need not be locally compact. [4]

(e) (i) Define path-connectedness. Prove that a path-connected space is connected. Is the converse true? Justify your answer. [1+3+3]

(ii) Prove that a real valued continuous function defined on a compact space is bounded and attains its least and greatest values. [3]