B.Sc. Semester V (Honours) Examination, 2021 (CBCS)

Subject: Physics

Paper: CC-XI

(Quantum Mechanics and Applications)

Time: 2 Hours **Full Marks: 40**

The questions are of equal value. Candidates are required to give their answers in their own words as far as practicable.

Answer any ***eight*** of the following questions $5 \times 8 = 40$

1. a) If a system has two eigenstates, ψ_1 and ψ_2 with eigenvalues E_1 and E_2, under what condition will the linear combination ($c_1\psi_1 + c_2\psi_2$) be also an eigenstate? (c_1 and c_2 are constants.)

   b) Find the eigenfunction of the operator $x + \dfrac{d}{dx}$.

2. a) A one-dimensional wave function is of the following wave form

   $$\psi(x,t) = Ae^{i\varphi(x,t)}$$

   Prove that the probability current density may be expressed as

   $$J = \frac{\hbar}{m}|A|^2 \frac{\partial \varphi}{\partial x}$$

   b) A particle on the x-axis has the wave function $\psi = Ax^2$ (A = constant) between $x = 0$ and $x = 2$. Find the probability that the particle can be found between $x = 0.5$ and $x = 0.6$.

3. Using Heisenberg's uncertainty principle find the first Bohr radius of hydrogen atom assuming proton to be at rest.

4. a) Show that the momentum operator $\hat{p}_x$ is Hermitian.

   b) Show that the commutator $\left[\hat{L}_x, \hat{y}\right]$ where $\hat{L}_x$ is the x-component of the angular momentum operator and $\hat{y}$ is the y-component of the position operator is equal to $i\hbar z$.

5. a) Show that the Gaussian wave function at $t = 0$,

   $$\psi(x) = \frac{1}{\left(\pi\sigma^2\right)^{1/4}} e^{-x^2/2\sigma^2} e^{ikx/\hbar}$$ is normalized.

   b) Evaluate the expectation value $\langle p_x \rangle$ for the above Gaussian wave function.

   (symbols have their usual meaning.)

6. A particle is confined in a one-dimensional potential well defined by

   $$V_x = \begin{cases} 0, & 0 < x < a \\ \infty, & x \le 0 \text{ and } x \ge a \end{cases}$$

   Obtain the energy eigenvalues and the normalized eigenfunction of the system.

7. A source of light is placed between the poles of an electromagnet. What will you observe if the light is examined by a spectroscope in directions parallel and perpendicular to the magnetic field? Assume the magnetic field to be not too strong. Give an explanation of the phenomenon.

8. a) For the atomic states with principal quantum number n = 3, what will be the maximum number of electrons in the $n = 3$, $l = 2$ and $l = 1$ states?

   b) If an atom is in the $^3D_3$ state, calculate the angle between its orbitals ($\vec{L}$) and spin angular momentum vector ($\vec{S}$).

9. a) What are the possible values of the quantum numbers j and m_j for a hydrogen atom in the $2p$ state?

   b) The normalized ground state wave-function of a hydrogen atom is given by

$$\psi_{100}(r,\theta,\varphi) = \frac{1}{\sqrt{\pi} a_0^{3/2}} \exp\left(-r/a_0\right)$$

   Calculate the average value of $(1/r)$, where r is the radial distance of the electron from the nucleus and a_0 is the radius of innermost Bohr orbit.

10. (a) The ground state of sodium atom is described by $3\,^2s_{1/2}$. Explain the meaning of this notation.

    (b) The first line of the Balmer series of hydrogen has a wavelength 656.3 nm. Calculate the wavelength of the second line.