B.A./B.Sc. 2nd Semester (Honours) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMH2CC04
(Differential Equation and Vector Calculus)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any ten questions: 10×2 = 20

(a) Show that the function e^x and xe^x are linearly independent on the x-axis. [2]

(b) What is an integrating factor of a first order and first-degree differential equation? Is it unique? Justify your answer. [2]

(c) Construct a differential equation having x and $sinx$ as its fundamental solution. [2]

(d) Apply Picard's theorem find the first two approximations of the initial value problem: $\frac{dy}{dx} = y - x$, with $y(0) = 1$. [2]

(e) Obtain two linearly independent solutions of the equation [2]

$$(D^2 + D - 2)y = 0, where\ D \equiv \frac{d}{dx}.$$

(f) Obtain three consecutive equilibrium points for the system [2]

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = sinx.$$

(g) Prove that the differential equation [2]

$(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$with $y(0) = 1$.

represents a family of hyperbolas.

(h) Find the solution of the differential equation [2]

$$\frac{dy}{dx} = y - sinx + cosx.$$

satisfying the condition that y should be bounded when x tends to infinity.

(i) Find a particular integral of [2]

$(D^2 - 2D)y = e^x sinx$, where $D \equiv \frac{d}{dx}$.

(j) Let *a(x)* and *b(x)* be two solutions of the differential equation [2]

$$\frac{dy}{dx} = y + 2022,$$

with the initial conditions $a(0) = 1, b(0) = 1$.

Then prove that *a(x)* and *b(x)* will never intersect .

(k) Find the projection of the vector $\vec{a} = 2\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 9\hat{\jmath}$. [2]

(l) Examine whether the three vectors $4\hat{\imath} + 2\hat{\jmath} + \hat{k}$, $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $4\hat{\imath} + 3\hat{\jmath} - \hat{k}$ are coplanar or not. [2]

(m) A force of 15 units acts in the direction of the vector $3\hat{j} - \hat{k}$ and passes through a point $\hat{\imath} + \hat{j} - \hat{k}$. Find the moment of the force about the point $4\hat{\imath} + 3\hat{j} - \hat{k}$. [2]

(n) Find the unit vector in the direction of the tangent at any point on the curve $\vec{r} = a\hat{\imath}\cos t + a\hat{j}\sin t + b\,t\,\hat{k}$, where a, b are constants. [2]

(o) Find a vector of magnitude 3 perpendicular to both the vectors $\hat{\imath} + 2\hat{j}$ and $4\hat{\imath} + \hat{j} + \hat{k}$. [2]

2. Answer any four questions: 4×5 = 20

(a) Obtain the power series solution of [5]
$(x^2D^2 + (x+1)D + 1)y = 0$, where $D \equiv \frac{d}{dx}$.
about the point $x = 0$.

(b) Solve: $(D^2 + 2)y = x^2e^{3x} + e^x \sin 2x$, where $D \equiv \frac{d}{dx}$. [5]

(c) Find a general solution of the system of equations [5]

$$(D+5)x + (D-3)y = 5$$
$$(D-1)x + Dy = t + 5,$$

where $D \equiv \frac{d}{dt}$.

(d) Find the family of curves for which the length of the part of the tangent between the point of contact *(x,y)* and the *y*- axis is equal to the *y*- intercept of the tangent. [5]

(e) If $\vec{A} = (2y + 4, xz + 8, yz - x)$, evaluate $\int \vec{A}.d\vec{r}$ along the curve c where c is given by $x = 2t^2$, $y = t$, $z = t^3$ from $t = 0$ to $t = 2$. [5]

(f) Show that the triple product $\vec{a}.(\vec{b}\times\vec{c})$ is a scalar which is numerically equal to the volume of the parallelopiped of which the three concurrent edges are $\vec{a}, \vec{b}, \vec{c}$. [5]

3. Answer any two questions: 2×10 = 20

(a) (i) Apply the method of variation of parameters to solve the equation [4]

$$(D^2 + 9)y = \sec 3x, \text{ where } D \equiv \frac{d}{dx}.$$

(ii) Solve: $xy(y - px) - x = py$ where $p = \frac{dy}{dx}$. [3]

(iii) Solve: $(D^2 + 4)y = \sin 2x + \cos 2x + 5$, where $D \equiv \frac{d}{dx}$. [3]

(b) (i) Using the method of undetermined coefficients solve: [4]

$$(D^2 + 4)y = 4\tan 2x, \text{ where } D \equiv \frac{d}{dx}.$$

(ii) Solve: $(D^2 + 2)(D^2 - 4)y = x^3e^{2x} + e^{-2x}\cos 2x$, where $D \equiv \frac{d}{dx}$. [3]

(iii) Find a general solution of the system of equations [3]

$$(D-9)x + (D+2)y = t^2e^t$$
$$(D-2)x + (D-4)y = t^2 - \log t,$$

where $D = \frac{d}{dt}$.

(c) (i) Solve: $(D^2 + 2D - 3)y = 9x$, with $y(0) = 1$, $y'(1) = 2$, where $D \equiv \frac{d}{dx}$. [5]

(ii) Consider the pair of first order ordinary differential equations [5]

$$\frac{dx}{dt} = -0\,.5x - 5y$$

$$\frac{dy}{dt} = x$$

with $x(0) = 0$, $y(0) = 1$. Then prove that $x(t)$ is bounded on $[0\,,\infty)$.

(d) (i) For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ show that [5]

$$\vec{a}.\left(\overrightarrow{b}\,\mathrm{x}\vec{c}\right) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}.$$

(ii) For the curve $\overrightarrow{r} = 2a\hat{\imath}\cos t + 2a\hat{\jmath}\sin t + t\,\hat{k}$, find $\left[\frac{d\vec{r}}{dt}\,\frac{d^2\vec{r}}{dt^2}\,\frac{d^3\vec{r}}{dt^3}\right]$. [5]