B.Sc. 4th Semester (Honours) Examination, 2022 (CBCS)

Subject: Physics

Paper: CC-VIII

(Mathematical Physics-III)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own word as far as practicable.

Group-A

1. Answer any *five* questions from the following: **2×5=10**

a) Using Cauchy's integral formula, evaluate the integral $\oint \frac{z^2}{z^2-1}\, dz$ around the unit circle with center at $z = 1$.

b) Show whether or not the functions i) $f(z) = z^2$ and $f(z) = z^*$ are analytic.

c) What do you mean by 'poles' and 'isolated singularities' of a function?

d) In which domain(s) of the complex plane is $f(z) = |x| - i|y|$ an analytic function?

e) If $f(s) = L\{F(t)\}$, then show that $L\{e^{at}F(t)\} = f(s-a)$. L is the Laplace transformation operator.

f) What is the Fourier transform of $\delta(t-a)$ where a is a constant.

g) If $\Phi(s)$ is the Fourier sine transform of $f(x)$ for $s > 0$ then show that $F_s\{f(x)\} = -\Phi(-s)$ for $s < 0$.

h) For any complex number z show that $|x| + |y| \le \sqrt{2}|x + iy|$.

Group-B

2. Answer any two questions from the following: $\mathbf{5 \times 2 = 10}$

a) Using the rule $L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(s)\, ds$, show that $\int_0^\infty \frac{sint}{t} dt = \frac{\pi}{2}$.

b) Find the Fourier Cosine transform of $f(x) = \frac{1}{1+x^2}$.

c) Verify Cauchy's theorem for the integral z^3 taken over the boundary of the rectangle with vertices $-1, 1, 1+i, -1+i$.

d) Using Parseval's identity evaluate the integral $\int_0^\infty \frac{dx}{(1+x^2)^2}$.

Group-c

3. Answer any two questions from the following: $\mathbf{10 \times 2 = 20}$

a) (i) Given $f(z) = \frac{1+z}{1-z}$. Find (a) $\frac{df}{dz}$ (b) determine where $f(z)$ is non- analytic.

(ii) State Residue theorem. Evaluate the integral

$$\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta}. \text{ (Given } a > |b|\text{)}$$ 3+1+6

b) Find the Laurent series of

$$f(z) = \frac{1}{z(z-2)^3}$$

about the singularities $z = 0$ and $z = 2$ (separately). Hence verify that $z = 0$ is a pole of order 1 and $z = 2$ is a pole of order 3 and find the residues of $f(z)$ at each pole.

Evaluate $\oint \frac{dz}{(z-a)^n}$, $n = 2,3,4,\cdots$ where $z = a$ is inside the simple closed curve . 3+3+4

c) A resistance R in series with inductance L is connected with e.m.f $\epsilon(t)$. The current i is given by

$$L\frac{di}{dt} + Ri = \epsilon(t)$$

If the switch is connected at $t = 0$ and disconnected at $t = a$, using Laplace transformation find the current i in terms of t. 10

d) (i) $F(s)$ and $G(s)$ be the Fourier transforms of $f(t)$ and $g(t)$ respectively. Using convolution property show that $f * g = g * f$.

(ii) Show that the Fourier transform of $\cos ax^2 = \frac{1}{\sqrt{2a}} \cos\left(\frac{s^2}{4a} - \frac{\pi}{4}\right)$. 3+7