B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Physics

Course : DSE-1(1)

(Advanced Mathematical Physics)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

All symbols have their usual meanings.

SECTION-I

1. Answer *any five* questions: 2×5=10

(a) The vectors $\vec{r}$ in real three dimensional space V_3 is transformed to $\vec{r}'$ by an operator A as follows: $\vec{r'} = A\vec{r} = \vec{a} \times \vec{r}$, where $\vec{a}$ is a constant vector. Show that A is linear.

(b) Define Hermitian matrix. Check whether the matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is Hermitian or not?

(c) What is the relation between the trace of a matrix and the eigenvalues of that matrix? What is the relation between determinant of a matrix and eigenvalues of that matrix?

(d) Write down Lorentz transformation equations in the context of Einstein's Special Theory of Relativity in matrix form.

(e) Define inner product of two tensors each of order one.

(f) If $A = \lambda x^i$ for all values of the independent variables $x^1, x^2, x^3, \ldots, x^n$. and λ_i's are constant, show that $\frac{\partial}{\partial x^j}(\lambda_i x^i) = \lambda_j$.

(g) Write down expression of the Christoffel 3 index symbols of the first and the second kind.

(h) Write down Quotient Law of tensors.

SECTION-II

(Answer *any two* questions.) 5×2=10

2. Linear transformation T on R^3 acts on standard basis and gives us ordered triplets of real numbers, defined by

$$T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix};\ T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}3\\1\\2\end{pmatrix}\ \textit{and}\ T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}2\\1\\3\end{pmatrix}$$

then compute $T\begin{pmatrix}3\\-1\\4\end{pmatrix}$

3. Exponential of a matrix can be written as power series of the given matrix (provided the series converges):

Using this hint prove that for a matrix $M = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, one can obtain $e^M = \begin{pmatrix} e^a & 0 \\ 0 & e^b \end{pmatrix}$

4. Show that in Rotational Mechanics moment of inertia (**I**) is actually a tensor of order 2. Here $\vec{L} = \vec{r} \times \vec{p} = \boldsymbol{I}\vec{\omega}$

5. (a) Show that all diagonal elements of an anti-symmetric matrix are zero.
   (b) Show that in an n-dimensional space S_n, an anti-symmetric (skew symmetric) covariant tensor of second order (i.e. anti-symmetric matrix) has at most $\frac{n(n-1)}{2}$ different components.

SECTION-III

(Answer *any two* questions.) 10×2=20

6. Solve the given coupled differential equations using matrix method:

$$\frac{dx}{dt} = 2x - 3y$$

$$\frac{dy}{dt} = y - 2x$$

Subject to initial conditions $x(0) = 8, y(0) = 3$. 10

7. Write down the Caley-Hamilton's theorem and using this theorem find the inverse of the matrix

$$\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$$ 10

8. Given $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2 \sin^2\theta(d\phi)^2$ defines the line element in a spherical polar coordinates. Find the values of [13, 3] and $\left\{ {3 \atop 13} \right\}$ 5+5=10

9. The line element is given

$$ds^2 = g_{pq}dx^p dx^q = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$$

Find the

(a) Conjugate tensor g^{ij}(also called second fundamental tensor) and
(b) g (determinant of g_{ij}) corresponding to the metric. 5+5=10

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