B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Physics

Paper : DSE-2(5)

(Classical Dynamics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols and abbreviations have their usual meanings.

1. Answer *any ten* of the following questions: 2×10=20

(a) Prove that the magnetic force does no work.

(b) Mention two advantages of Lagrangian mechanics over Newtonian mechanics.

(c) A double pendulum consists of two point masses m attached by strings of length l. The strings make angles θ_1 and θ_2 with the vertical axis, find the kinetic energy of the pendulum.

(d) If the Lagrangian of a particle moving in one dimension is given by $L = \frac{\dot{x}^2}{2m} - V(x)$, find the corresponding Hamiltonian.

(e) If the Hamiltonian does not depend upon time explicitly, show that it is a conserved quantity.

(f) A particle of mass m is moving in the potential $V = -\frac{a}{2}x^2 + \frac{b}{4}x^4$ where a and b are positive constants. Find the equilibrium position of the particle about which the small oscillation is observed.

(g) Define normal coordinates and normal frequencies.

(h) State the postulates of special theory of relativity.

(i) Two events separated by a (spatial) distance 9×10^9m are simultaneous in one inertial frame. What will be the time interval between these two events in a frame moving with a constant speed 0.8 c (where the speed of light $c = 3 \times 10^8$m/s).

(j) What is world line? Draw the world line in S-frame of a particle at rest in S′inertial frame where S' frame moves with velocity $\vec{v}$ with respect to S-frame along x-direction.

(k) What is future and past light cone in special theory of relativity?

(l) An inertial frame S′ moves with velocity $\vec{v}$ relative to another inertial frame S along x-axis. Obtain the value of length in S which is unit length in S′.

(m) The muon has rest mass 105 MeV/c^2 and energy 315MeV. Find the velocity of muon.

(n) What is Reynold's number? State its significance.

(o) Define streamline and turbulent motion of fluid flow.

2. Answer *any four* of the following questions: 5×4=20

(a) Define generalised coordinates of a dynamical system. Obtain the Lagrangian and Lagrange's equation of motion of simple pendulum. Find the angular frequency for small amplitude of the oscillation. 1+3+1=5

(b) What do you mean by rotational invariance of a quantity? If the Lagrangian of a closed system remains rotationally invariant, prove that the angular momentum of the system is a conserved quantity. 1+4=5

(c) Establish the Lagrangian and deduce the Lagrange's equations of motion for small oscillations of a system with N degrees of freedom in the neighbourhood of stable equilibrium. 5

(d) Write down the Lorentz transformations in Minkowski space (x, y, z, t). Explain the time dilation using Minkowski diagram. 2+3=5

(e) Explain the term 'two events with space-like separation has no causal relation'. Derive the four acceleration vector in terms of three acceleration vector and three velocity vector of the particle. 2+3=5

(f) Using Naiver-Stokes equation for incompressible fluid derive the velocity-profile of fluid-flow in a cylindrical pipe. 5

3. Answer *any two* of the following questions: 10×2=20

(a) (i) A crossed electric and magnetic field is applied to a region where a charged particle of mass m and charge q is at rest at the origin. Derive the equation of motion of the charged particle.

(ii) Find the eigen frequencies of small oscillations of a system containing two equal masses attached by a spring. Also find the relation between the amplitudes of the two masses at the eigen frequencies. 6+(3+1)=10

(b) (i) Define Hamiltonian of a system. Obtain the Hamilton's equations of motion.

(ii) A particle of mass m is constrained to move on a cylindrical surface of radius R under potential $V = \frac{1}{2}k(R^2 + z^2)$ where k is a constant and z-axis is the axis of the cylinder. Construct the Hamiltonian and Hamilton's equations of motion. Show that the particle oscillates along z-axis. (1+3)+(5+1)=10

(c) (i) Show that Lorentz transformation is an imaginary rotation of an orthogonal coordinate system.

The space-time coordinates (x,y,z,t) of two events in S-frame are (0,0,0,0) and (6c,0,0,4), where c is the speed of light in vacuum and the time coordinate is in second. Find the space-time interval between two events. Mention the nature of the interval.

(ii) Define four-momentum (P_μ) vector. Show that it is related with three momentum vectors and energy. Prove that $P_\mu P^\mu$ is a Lorentz invariant. (3+2)+(1+2+2)=10

(d) (i) Prove that $x_\mu x^\mu$ is an invariant quantity under Lorentz transformation.

A nucleus at rest with mass m_0 decays spontaneously into two components of rest masses m_1 and m_2. Show that $m_0 > m_1 + m_2$. Explain by considering energy-momentum conservation.

(ii) What do you mean by fluid particle? Derive the equation of continuity for fluid flow.

(2+4)+(1+3)=10
