B.A/B.Sc.6th Semester (Honours) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMH6DSE41
(Bio Mathematics)

Time:3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any ten questions 10×2 = 20

(a) What is the drawback of Malthus growth? [2]

(b) What is a half saturation constant in Michaelis-Menten kinetics? [2]

(c) Explain the term 'harvesting' in a single population model. [2]

(d) What do you mean by competition in population? [2]

(e) Identify the activator and inhibitor form of the following system [2]

$$\frac{du}{dt} = a + \frac{bu}{c+v} - eu,$$
$$\frac{dv}{dt} = u - hv.$$

(f) What is insect out break? Give an example. [1+1]

(g) Define equilibrium point of a system. [2]

(h) What do you mean by a phase plane? [2]

(i) What is bifurcation? [2]

(j) Define Lotka-Volterra model model with diffusion. [2]

(k) Find the order of the following difference equation [2]

$$x_t - 4x_{t-1} + 4x_{t-2} = t^2.$$

(l) How do you define a steady state of a discrete system [2]

$$x_t = f(x_{t-1})?$$

(m) Find the general solution of [2]

$$x_{t+1} = ax_t,$$

with the initial condition $x_0 = k$.

(n) Give an example of a discrete prey-predator model. [2]

(o) Find the equilibrium point of $\dot{x} = x(x-1)$ [2]

2. Answer any four questions 4×5 = 20

(a) What is a chemostat? Define a model for bacterial growth in chemostat. [2+3]

(b) Define a simple SIS epidemic model and hence define the basic reproduction number from it. [2+3]

(c) Consider the following prey-predator model [5]

$$\frac{dH}{dt} = H\left(1 - \frac{H}{K}\right) - a\frac{PH}{b+H},$$
$$\frac{dP}{dt} = cP\left(-1 + g\frac{H}{b+H}\right),$$

where H, P are the prey and the predator respectively and all the parameters are positive. Find the corresponding dimensionless form.

(d) Obtain the Routh-Hurwitz criteria for a cubic monic polynomial to have all real roots. [5]

(e) Solve the following initial value problem by the method of characteristics [4+1]

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0, -\infty < x < \infty, 0 < t < \infty,$$
$$u(0,x) = \phi(x), -\infty < x < \infty.$$

Hence define travelling wave solution.

(f) (i) For a cubic characteristic polynomial$p(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$, state the jury conditions when the roots λ_i $(i = 1,2,3)$ satisfy $|\lambda| < 1$. [3]

(ii) Verify whether the jury conditions are satisfied or not for the following characteristic polynomial [2]

$$\lambda^3 - 0.25\lambda^2 - 0.25\lambda + 0.75.$$

3. Answer any two questions 2×10 = 20

(a) Discuss the phase plane analysis of the following two-dimensional system

$$\frac{dx}{dt} = ax + by,$$
$$\frac{dy}{dt} = cx + hy,$$

when

(i) the eigenvalues are real and [5]

(ii) the eigenvalues are complex conjugate. [5]

(b) Consider the following Lotka-Volterra competitive system

$$\frac{dx}{dt} = x(4 - x - y),$$
$$\frac{dy}{dt} = y(8 - 3x - y).$$

(i) Find all the equilibrium points of the system. [4]

(ii) Determine the stability of each equilibrium and state whether the equilibrium is a node or spiral. [6]

(c) (i) Reduce the following two species diffusion model into a linearized system around [6] any specially-uniform steady state

$$\frac{dN_1}{dt} = f(N_1, N_2) + D_1 \frac{\partial^2 N_1}{\partial x^2},$$
$$\frac{dN_2}{dt} = g(N_1, N_2) + D_2 \frac{\partial^2 N_2}{\partial x^2}.$$

(ii) Finally state the conditions for diffusive instability of the system. [4]

(d) Consider the following Nicholson-Bailey model

$$N_{t+1} = rN_t e^{-aP_t},$$
$$P_{t+1} = cN_t(1 - e^{-aP_t}).$$

(i) Find the interior equilibrium point. [3]

(ii) Prove that the interior equilibrium point is unstable if $r > 1$. [7]

B.A/B.Sc. 6th Semester (Honours) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMH6DSE42
(Differential Geometry)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any ten questions 10×2 = 20

(a) State Euler's theorem on a surface.

(b) Give an example of a surface of negative curvature. [2]

(c) Give an example of a compact surface. [2]

(d) Give an example of a surface of which every point is umbilic. [2]

(e) Define Gaussian curvature of a surface. [2]

(f) Define developable surface. [2]

(g) Is quadric always represents a surface? Support your answer. [2]

(h) Define geodesic curvature of a curve on surface. [2]

(i) Define normal curvature of a curve on a surface. [2]

(j) Write the expression among three fundamental forms of a surface. [2]

(k) Is double cone represents a smooth surface? Justify your answer. [2]

(l) Give an example of a flat surface. [2]

(m) Prove that any part of a straight line on a surface is a geodesic. [2]

(n) What are the geodesics on a sphere? [2]

(o) Prove that any normal section of a surface is a geodesic. [2]

2. Answer any four questions $4\times5 = 20$

(a) Deduce the curvature of a circular helix. [5]

(b) Obtain a necessary and sufficient condition for a space curve to be a plane curve. [5]

(c) Deduce the first fundamental form of a unit sphere. [5]

(d) Define a smooth surface. Prove that a plane is a smooth surface. [2+3]

(e) Find out the principal curvatures of a right circular unit cylinder. [5]

(f) Deduce Rodrigues formula for a curve lying on a surface. [5]

3. Answer any two questions $2\times10 = 20$

(a) Deduce Serret Frenet formulae for a space curve. [10]

(b) State and prove fundamental theorem of a plane curve. [10]

(c) Deduce the Gaussian curvature of a surface of revolution. [10]

(d) Using the geodesic equations, deduce the geodesics on the unit sphere. [10]

B.A/B.Sc. 6th Semester (Honours) Examination, 2022 (CBCS)
Subject: Mathematics
Course: BMH6DSE43
(Mechanics-II)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meanings]

1. Answer any ten questions $10\times2 = 20$

(a) What is meant by forces of constraint? [2]

(b) "Any frame of reference rotating uniformly with respect to an inertial frame is not an inertial frame." Justify your answer. [2]

(c) State Archimedes' principle for a floating body. [2]

(d) Explain first law of thermodynamics. [2]

(e) Define the normal and shearing stress at a point in a continuous medium. [2]

(f) Write down the stress matrix at a point in an ideal fluid, explaining the [2] symbols used.

(g) Does the velocity of light remain invariant under Galilean transformation? Justify your answer. [2]

(h) Write down the conditions of equilibrium for a freely floating body. [2]

(i) A plane lamina is slowly lowered in a liquid at rest. How will its centre of pressure and centre of mass change in the lamina when it is lowered (i)vertically , (ii) horizontally?

(j) What do you mean by the surface of buoyancy and the curve of buoyancy? [2]

(k) Define absolute time. [2]

(l) Can a homogeneous fluid be compressible? Explain your answer. [2]

(m) Is Lagrangian of a system unique? Justify your answer. [2]

(n) What do you mean by convective equilibrium? [2]

(o) Define scleronomic constraint. Give an example of it. [2]

2. Answer any four questions $4 \times 5 = 20$

(a) (i) Discuss briefly the Galilean transformation. [2]

(ii) A clock in a spaceship moving with a velocity (4c/5), where c is the velocity of light, given a time interval of 10 minutes between two events at a fixed place. Calculate the time between the same events as measured by an observer on earth. [3]

(b) Prove that if the density of a liquid at rest under gravity varies as the square root of the pressure, the density increases uniformly with the depth. [5]

(c) A thin uniform rod of weight **W** is loaded at one end with a weight **P** of negligible volume. If the rod float at any inclination with $\frac{1}{n}$th of its length out of the water, prove that **(n-1)P = W** . [5]

(d) Establish the relation between pressure and density at a height in an isothermal atmosphere , when gravity is not constant. [5]

(e) (i) Define Stress Quadric of Cauchy. [2]

(ii) Determine Cauchy's stress quadric for the following state of stress [3]

$$(\tau_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

(f) A circular hoop rolls down a rough inclined plane. Find the Lagrangian function for the motion. [5]

3. Answer any two questions 2×10 = 20

(a) (i) Deduce the differential equation of pressure for a fluid at rest under the action of external forces. [3]

(ii) State and prove the necessary and sufficient condition for equilibrium of a fluid under the action of external forces. [2+5]

(b) (i) A given volume V of heavy liquid is at rest under the action of forces $-\mu x$, $-\mu y$, $-\mu z$ [$\mu > 0$, a constant] Find the equation of the free surface. [5]

(ii) An equilateral triangular lamina ABC suspended freely from A, rests with the side AB vertical and the side AC bisected by the surface of a heavy liquid. Prove that the ratio of the density of the lamina to that of the liquid is **15:16.** [5]

(c) (i) A gas satisfying Boyle's law $p = k\rho$ is acted upon by the force $X = -\frac{y}{x^2+y^2}$, $Y = \frac{x}{x^2+y^2}$. Find the density for adiabatic expansion of a compressible fluid, where the symbols are to be Explained by you. [5]

(ii) Define absolute length. [2]

(iii) Show that the distance between two points remains invariant under Galilean transformation. [3]

(d) (i) Define a holonomic bilateral constraint with example. [3]

(ii) Deduce Lagrange's equations of motion for a holonomic system of **N** particles with **k** bilateral constraints. [7]