B.Sc. 6th Semester (Honours) Examination, 2022 (CBCS)
Subject: Physics
Paper: CC-XIII
(Electromagnetic Theory)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own word as far as practicable.

Group-A

1. Answer any *five* questions from the following: **2×5=10**

a) The magnetic field in a plane electromagnetic wave is given by

$$B_y = (2\times10^{-7}T)\sin(0.5\times10^{3}x + 1.5\times10^{11}t)$$

Write an expression for the electric field and calculate the frequency of the wave.

b.) Show that the displacement current in the dielectric of a parallel plate capacitor

is equal to the conduction current in the connecting leads.

c.) Show that the wave equation in free space can be written in the form

$(\nabla^2 + k^2)\vec{E} = 0$ where k is the wave vector.

d) Explain the behaviour of water to electromagnetic waves in water of frequencies 200 Hz

and 100 MHz, when the dielectric constant of water is 80 and $\sigma = 10^{-3}\Omega^{-1}m^{-1}$.

e) Show that for any set of scalar and vector potentials representing a particular EM field,

we can always find another set of potentials representing the same field.

f) Relative refractive index difference for an optical tube designed for long distance transmission is 20%.Estimate the numerical aperture and critical angle at core-cladding interface with the fibre when core refractive index is 1.46.

g) Show that an excess charge placed at any point in a medium of conductivity σ and permittivity $\in$ decays exponentially with a time constant $\frac{\in}{\sigma}$.

h) The specific rotation of the quartz for λ = 508.6 nm is 29.73 degree per minute .Calculate the difference between the refractive indices for left and right circularly polarized light for quartz.

Group-B

2. Answer any two of the following questions. $5\times 2=10$

a.) i) Obtain Kirchhoff's voltage law for series LCR circuit u sing Maxwell's equation.

ii) An e.m. wave of wavelength λ_0 propagates in vacuum from the point A to the point B.

If a dielectric slab of thickness d (d<AB) is introduced in the path, show that the change in phase $\Delta\phi$ of the wave is given by $\Delta\phi=\frac{2\pi d}{\lambda_0}(\sqrt{K}-1)$, where is K dielectric constant of the slab. $2\frac{1}{2}+2\frac{1}{2}$

b.) Show that for a plane electromagnetic wave propagating in a good conductor, the field energy is almost entirely magnetic in character. 5

c) With appropriate formula explain how can you convert a left-handed elliptically polarized light into a right-handed one? How will you detect experimentally a plane polarized light from a partially polarized light (experimentally). 4+1

d) Show that a plane polarized light can be considered as the superposition of two circularly polarized waves rotating in opposite directions. 5

Group-C

3. Answer any two of the following questions $10\times 2=20$

a) i) Show that under suitable conditions vector potential $\vec{A}$ and scalar potential ϕ satisfy the following inhomogeneous equations : $\nabla^2 A-\varepsilon\mu\frac{\partial^2 A}{\partial t^2}=-\mu\vec{J}$ and $\nabla^2\phi-\varepsilon\mu\frac{\partial^2\phi}{\partial t^2}=-\frac{\rho}{\varepsilon}$ where the symbols have their usual meanings.

ii) Show that the electromagnetic potentials in uniform electric and magnetic field can be expressed as

$\phi=-\vec{E}.\vec{r}$ and $\vec{A}=\frac{1}{2}(\vec{B}\times\vec{r})$. $(3\frac{1}{2}+3\frac{1}{2})+(1+2)$

b) i) Consider the propagation of electromagnetic waves through a dilute ionized gas. Show that it behaves like a medium of refractive index $n=\sqrt{1-\frac{\omega_p^2}{\omega^2}}$ where ω_p is the electron-plasma frequency and ω is the angular frequency of the wave. Hence show that the critical frequency below which wave propagation through it is not possible is given by $f_c \cong 9\sqrt{n_0}$ where n_0 is the number of electrons per meter^3.

ii) In free space electric field associated with an electromagnetic wave is given by $\vec{E} = E_m \sin(\omega t - kz)\hat{j}$. Using Maxwell's equations, find the corresponding magnetic field $\vec{H}$. Graphically represent $\vec{E}$ and $\vec{H}$ fields at t=0. $(5+2)+(2+1)$

c) Consider the propagation of electromagnetic waves between two parallel perfectly conducting plates. Considering TE mode show that the waves can propagate only if the wavelength is smaller than a certain critical value λ_c for a particular mode. If λ_g is the guide wavelength and λ_0 is the free space wavelength, then show that $\frac{1}{\lambda_0^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$. Show that the phase velocity is greater than the speed of light in vacuum and interpret the result. Show that $v_p v_g = c^2$ where $v_p =$ phase velocity and $v_g =$ group velocity. 4+2+2+2

d) i) A plane polarized EM wave falls obliquely at the boundary of two dielectrics. The plane of polarization of the wave is perpendicular to the plane of incidence. Show that there is always a phase change of π in the reflected wave for reflection at the interface backed b y denser medium, but there is no such phase change if the interface is backed by rarer medium. Also, show that there is no Brewster's angle here as long as the media have different refractive indices assuming the expression for amplitude reflection coefficient.

ii) A plane polarized EM wave with its electric vector making an angle of 45^0 with the plane of incidence, is incident at an angle θ on an interface of two dielectrics of refractive indices n_1 and n_2 .Show that the electric vector of the wave transmitted into medium 2 makes an angle ϕ with the plane of incidence, which is given by $\phi = \tan^{-1}[\cos\{\theta - \sin^{-1}(\frac{n_1}{n_2}\sin\theta)\}]$

(Assume the expressions for the amplitude transmission coefficients).

iii) Find the value of transmission coefficient when an electromagnetic wave is incident normally on the glass-air interface ($n_1 = 1.5$ and $n_2 = 1.0$). 5+ 4+1