

B.Sc. 1st Semester (Honours) Examination, 2022 (CBCS)**Subject : Physics****Course : CC-I****Time: 2 Hours****Full Marks: 40***The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**Abbreviations and symbols have their usual meaning.***Group-A**

Answer any five questions.

2×5=10

1. (a) Describe the locus of a particle whose position vector is $\vec{r}(\theta, \varphi) = (a \cos \theta \cos \varphi)\hat{i} + (b \sin \theta \cos \varphi)\hat{j} + (c \sin \varphi)\hat{k}$.
- (b) Find the normal vector and equation of the tangent plane to the surface $z = \sqrt{x^2 + y^2}$ at (3, 4, 5).
- (c) A water tank having height H and cross-sectional area A is completely filled. There is an orifice of cross-sectional area α at the bottom. Calculate the time the tank will be empty when the orifice is opened.
- (d) Find the directional derivative of $\Phi(x, y, z) = x^3 + y^3 + z^3$ at (1, -1, 2) in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$.
- (e) Find the unit vector \hat{e}_r and \hat{e}_θ in polar co-ordinate system.
- (f) Find the Taylor series expansion of e^x about $x = 0$.
- (g) Two players A and B play a game such that A has probability $\frac{2}{3}$ of winning. If A plays 4 games, find the probability that A wins exactly 2 games.
- (h) Evaluate the integral $\int (x^3 + 1) \delta(x^2 - 9) dx$.

Group-B

Answer any two questions.

5×2=10

2. (a) Show that $\int_C \frac{xdx+ydy}{\sqrt{x^2+y^2}}$ is independent of the path of integration which does not pass through the origin. Find the value of the integral from the point $P(-1, 2)$ to $Q(2, 3)$. 2+3
- (b) Solve the differential equation:

$$\frac{dy}{dx} - y = y^2(\sin x + \cos x)$$

- (c) Find the expression of $\vec{\nabla} \cdot \vec{V}$ in curvilinear co-ordinate system. Hence write the expression for spherical co-ordinates. 4+1

- (d) If $f(x) = \frac{1}{2}(x+1)$ for $-1 < x < 1$ and
 $= 0$ otherwise

is a density function, calculate $E(x)$ and $Var(x)$.

2+3

Group-C

Answer any two questions.

10×2=20

3. (a) Evaluate the integral $\int_C [(x^2 - y^3)dx + (x + y)dy]$, where C is the contour bounded by $y = x$, $y = -x$ and $x^2 + y^2 = 16$ for $x \geq 0$.
 (b) Solve the differential equation:

5+5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

4. (a) A boat is rowed with a velocity u directly across a stream of width a . If the velocity of the current is directly proportional to the product of distances from the two banks, find the path of the boat and the distance where it will land.
 (b) Evaluate $\iint \Phi \hat{n} ds$ where $\Phi = \frac{3}{8}xyz$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

5+5

5. (a) Represent the vector $\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in spherical co-ordinates.

- (b) Prove that $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$.

5+5

6. (a) Prove that Poisson distribution is the limiting case of binominal distribution for very large value of n (no. of trials) and small value of p (probability of occurrence) such that $n.p$ is finite and equals to λ .

- (b) A set of observations is as:

$x:$ 0 1 2 3 4

$f:$ 30 63 45 10 2 where f = frequency

Construct the distribution function. Hence calculate the theoretical frequency for the third observation ($x = 2$). Given $e^{-1.3} = 0.273$.

6+2+2