# B.A./B.Sc. 3rd Semester (Honours) Examination, 2023 (CBCS)

## Subject : Mathematics

## Course : BMH3CC07

## (Numerical Methods)

**Time : 2 Hours** **Full Marks : 40**

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*Notation and Symbols have their usual meaning.*

**1.** Answer *any five* questions: 2×5=10

(a) If $u(x,y,z) = xyz^2$ and errors in $x, y, z$ are $0{\cdot}005$, $0{\cdot}001$ and $0{\cdot}002$ respectively at $x = 3$, $y = 1$, $z = 1$. Compute the maximum absolute error in evaluating $u$ at $(3,1,1)$.

(b) Find the number of significant figure in $V_T = 1{\cdot}5923$ given its relative error as $0{\cdot}1 \times 10^{-3}$.

(c) If $f(x) = x^2$, then show that $\Delta^r f(x) = 0$ for $r \geq 3$, where $\Delta$ is the forward difference operator.

(d) Write down the geometric interpretation of modified Euler's method.

(e) Find the values for $x_1$, $x_2$ and $x_3$ while solving the equations

$4x_1 + x_2 + 2x_3 = 4$

$x_1 + x_2 + 3x_3 = 3$

$3x_1 + 5x_2 + x_3 = 7,$

by Gauss-Seidal iterative method after one iteration on taking $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$.

(f) What do you mean by order of convergence of an iterative method? What is the order of convergence of Regula-Falsi method? 1+1=2

(g) What are the advantages and disadvantages of partial and complete pivoting?

(h) Find the degree of precision of Simpson's one-third rule.

**2.** Answer *any two* questions: 5×2=10

(a) The equation $x^2 + ax + b = 0$ has two real roots α and β. Show that the iteration method $x_{k+1} = -\frac{ax_k+b}{x_k}$ is convergent near $x = \alpha$ if $|\alpha| < |\beta|$ and $x_{k+1} = -\frac{x_k^2+b}{a}$ is convergent near $x = \alpha$ if $2|\alpha| < |\alpha + \beta|$.

(b) Solve the following linear system of equations by Gauss-Jordan method:

$4x_1 - 2x_2 + x_3 = -8$

$3x_1 + 9x_2 - 2x_3 = 11$

$4x_1 + 2x_2 + 13x_3 = 24$

(c) If $u_x = a + bx + cx^2$, prove that $\int_1^3 u_x\, dx = 2u_2 + \frac{1}{12}(u_0 - 2u_2 + u_4)$ and hence find an approximate value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-\frac{x^2}{10}} dx$. 3+2=5

(d) Describe power method for determination of the largest eigen value and the corresponding eigen vector of a square matrix. When does the method fail? 4+1=5

**3.** Answer *any two* questions: 10×2=20

(a) (i) Deduce Lagrange's interpolation formula from Newton's divided difference interpolation formula. 5

(ii) Complete the following table: 5

| $x$ | 10 | 15 | 20 | 25 | 30 | 35 |
|---|---|---|---|---|---|---|
| $f(x)$ | 19·97 | 21·51 | – | 23·52 | 24·65 | – |

(b) (i) Explain the method of fixed-point iteration with the condition of convergence for numerical solution of an equation of the form $x = \phi(x)$. 5

(ii) Show that the Cote's co-efficients $K_r^{(n)}, r = 0, 1, 2, \ldots, n$ in Newton-Cote's quadrature formula satisfy the relation $\sum_{r=0}^{n} K_r^{(n)} = 1$. 5

(c) (i) Describe the composite Weddle's rule of integration.

(ii) Using Newton's forward interpolation formula obtain the expression of $f'(x)$. 5+5=10

(d) (i) Solve the following system of equations by LU-decomposition method:

$x_1 + x_2 - x_3 = 2$

$2x_1 + 3x_2 + 5x_3 = -3$

$3x_1 + 2x_2 - 3x_3 = 6$

(ii) Establish the second-order Runge-Kutta method for solving the differential equation $\frac{dy}{dx} = f(x, y)$ subject to the condition $y(x_0) = y_0$. 5+5=10

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