B.Sc. 3rd Semester (Honours) Examination, 2023 (CBCS)

Subject : Physics

Course : CC-V

(Mathematical Physics)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as applicable.

1. Answer *any five* questions from the following: 2×5=10

   (a) What is Rodrigue formula for Legendre polynomial?

   (b) Check the orthogonality of $\sin 3x$ and $\sin 4x$ in the interval $-\pi$ to π.

   (c) Write down the Laguerre differential equation explaining the significant terms. Comment on the singularity at $x = 0$.

   (d) Show that if a real $f(x)$ is expanded in a complex exponential Fourier series $\sum_{-\infty}^{\infty} C_n e^{inx}$, then $C_{-n} = \overline{C_n}$, where $\overline{C_n}$ means the complex conjugate of C_n.

   (e) Find the value of $\Gamma(2/3)/\Gamma(8/3)$.

   (f) Find the value of $erf(\infty)$.

   (g) Express x^3 as a linear combination of Legendre polynomials.

   (h) State Fuch's theorem.

Answer *any two* questions from the following. 5×2=10

2. (a) Prove that $\int_0^1 x\, J_n(\alpha x) J_n(\beta x) dx = \frac{1}{2}\, [J_{n+1}(\alpha)]^2$ if $\alpha = \beta$.

   (b) What do you mean by orthogonality of Bessel functions? 4+1

3. (a) Assume separation of $u = R(r)\Theta(\theta)\Phi(\varphi)$. Hence show that Laplace's equation can be decomposed into three total differential equations.

   (b) Find the solution of Φ-equation subject to the condition $\Phi(\varphi + 2\pi) = \Phi(\varphi)$. 4+1

4. Express $\displaystyle\int_0^1 \frac{x^4}{\sqrt{1-x^2}}\, dx$ in terms of β-function and evaluate. 3+2

5. Prove that $\frac{1}{2} + \sum_{n=1}^{n} \cos nx = \dfrac{\sin\left(n+\frac{1}{2}\right)x}{2\sin\left(\frac{x}{2}\right)}$. 5

Answer *any two* questions from the following. 10×2=20

6. (a) Solve the following equation using the method of series solution: $x^2 y'' - 3xy' + 3y = 0$.

   (b) Show that $\int_{-1}^{1} x^m\, P_n(x) dx = 0$ if $m < n$. 5+5

7. (a) Consider the function $f(x) = x^2, -\pi \leq x \leq \pi$.
Plot $f(x)$ in the interval $-\pi$ to π and also plot its periodic extension upto -2π and 2π.

(b) Find out the Fourier expansion of $f(x)$.

(c) Prove that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

(1+1)+6+2

8. (a) Prove that $\iint_D x^{l-1} y^{m-1} \, dxdy = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m+1)} h^{l+m}$ where D is the domain $x \geq 0, y \geq 0$ and $x + y \leq h$.

(b) Show that $\frac{d}{dx}[erf(ax)] = \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$.

(c) Considering the frequency distribution of error as $f(x) = Ae^{-h^2x^2}$, show that average error is $\eta = \frac{1}{h\sqrt{\pi}}$.

4+3+3

9. A rectangular stretched membrane of sides a and b having its edges parallel to x– and y–axes, and bounded rigidly at the edges, is given a slight deformation in the z-direction perpendicular to the plane. The differential equation for z is given by $\frac{\partial^2 z}{\partial t^2} = c^2 \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$, where c is a constant. Solve the equation by the method of separation of variables, assuming the initial conditions $z = f(x, y)$ and $\frac{dz}{dt} = 0$ at $t = 0$.

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