

B.Sc. 4th Semester (Honours) Examination, 2023 (CBCS)**Subject : Physics****Course : CC-VIII****Time: 2 Hours****Full Marks: 40***The questions are of equals value.**The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.***Group-A****1. Answer any five questions:****2×5=10**

- (a) Consider the function : $w = \tanh^{-1} z$. Prove that the principal branch of the function leads to $\tanh^{-1} z = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right)$.
- (b) Express $\frac{1+3i}{1-2i}$ in the form $re^{i\theta}$.
- (c) Prove that $\sin(ix) = i \sinh x$.
- (d) Evaluate $\oint_C (z-a)^n dz$ where C is the circle at centre a and of radius r . What will be the result of the integration for $n = -1$.
- (e) Find the Fourier sine transform of $\left(\frac{1}{x}\right)$.
- (f) If $F(S)$ is the Fourier transform of $f(x)$, prove that $Ff(ax) = \frac{1}{a} F\left(\frac{S}{a}\right)$.
- (g) Prove that $L(t^n) = \frac{n!}{S^{n+1}}$ where L stands for Laplace transform and both n and S are +ve.
- (h) Express $\sin^5 \theta$ in terms of sines of odd multiples of θ .

Group-B**2. Answer any two questions:****5×2=10**

- (a) Determine the poles of the function:

1+4

$$f(z) = \frac{1}{z^4 + 1}$$

- (b) What do you mean by analyticity of a function $f(z)$? Find the analytic function $f(z)$ such that $\operatorname{Re} f'(z) = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$.

1+4

(c) Find the Laplace transform of $f(t) = \sin 2t \cos 3t$. 5

(d) Write the Fourier cosine transform of a function $f(x)$. Determine $f(x)$ from the relation: 2+3

$$\int_0^{\infty} f(x) \cos \omega x dx = e^{-\omega}$$

Group-C

Answer any two questions.

10×2=20

3. (a) Write Cauchy integral formula. Evaluate the integral:

$$\oint_C \frac{\sin^6 z}{\left(z - \frac{\pi}{6}\right)^3} dz \text{ where } C \text{ is the circle } |z| = 1, \text{ using Cauchy integral formula.} \quad 2+3$$

(b) Write Residue theorem. Evaluate the integral $\oint_C \frac{e^z}{(z^2 + \pi^2)^2} dz$ where C is the circle $|z| = 4$.

2+3

4. (a) Write Laurent's theorem for an analytic function. Expand the function in Laurent's series for, $1 < |z| < 3$ and $f(z) = \frac{1}{(z+1)(z+3)}$. 2+4

(b) Check whether the function $f(z) = \log z$ for $z > 0$ is analytic or not. 4

5. (a) Evaluate $\int_0^{\infty} t e^{-3t} \sin t dt$. 3

(b) Find $L[f'(t)]$ where L stands for Laplace transform. 2

(c) Solve the equation: $y''(t) + 9y(t) = 6 \cos 3t$ using Laplace transform. Given: $y(0) = 2$ and $y'(0) = 0$. 5

6. (a) Find the Fourier transform of $f(x)$ defined as:

$$\begin{aligned} f(x) &= x^2, \quad |x| < a \\ &= 0, \quad |x| > a \end{aligned} \quad 5$$

(b) Find the complex form of Fourier integral representation of $f(x)$ defined as:

$$\begin{aligned} f(x) &= e^{-kx}, \quad x > 0, \quad k > 0 \\ &= 0 \text{ otherwise} \end{aligned} \quad 5$$