B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH5DSE11

(Linear Programming)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Define basic feasible solution of an LPP.

(b) What do you mean by convex hull and convex polyhedron?

(c) Is the following set A in R^2 is convex? Justify with your reason. 1+1
$A = \{(x, y): x > 0, y > 0 \text{ and } xy \leq 1\}$

(d) Find the extreme points of the set $A = \{(x, y): |x| \leq 2, |y| \leq 2\}$.

(e) What do you mean by alternative optima of an LPP?

(f) Find a basic feasible solution of the following system of equations:

$$x_1 + 4x_2 - x_3 = 3$$
$$5x_1 + 2x_2 + 3x_3 = 4$$

(g) Test whether the following set of vectors are linearly dependent or not.
$\{(3,0,2), (7,0,9), (4,1,2)\}$

(h) Find the condition under which the following game problem will be a fair game.
$\begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$ where a, b, c, d are all ≥ 0.

(i) Determine the value of θ so that the game with following payoff matrix is strictly determinable.

	Player B		
Player A	θ	6	2
	− 1	θ	− 7
	−2	4	θ

(j) Give an example of symmetric game and find its value.

(k) Prove that the solution of a transportation problem with 2 origins and 3 destinations is bounded. 1+1

(l) Find the dual of the primal problem given by

Minimize $Z = -6x_1 - 8x_2 + 10x_3$

subject to

$x_1 + x_2 - x_3 \geq 2,$

$2x_1 - x_3 \geq 1,$

$x_1, x_2, x_3 \geq 0.$

(m) State complementary slackness theorem.

(n) "All boundary points are not necessarily extreme points."— Justify this statement with example.

(o) Prove that if a linear programming problem has two feasible solutions, then it has an infinite number of feasible solution.

2. Answer *any four* questions: 5×4=20

(a) Use Simplex method to obtain inverse of the matrix $\begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix}$.

(b) Solve the following linear programming problem:

Maximize $Z = 3x_1 + 4x_2$

Subject to $x_1 - x_2 \geq 0,$

$-x_1 + 3x_2 \leq 3$

and $x_1, x_2 \geq 0,$

(c) Use Dual simplex method to solve the LPP:

Minimize $Z = 10x_1 + 6x_2 + 2x_3$

subject to $-x_1 + x_2 + x_3 \geq 1,$

$3x_1 + x_2 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0.$

(d) Use dominance property to reduce the following payoff matrix to 2 × 2 matrix and hence solve the problem:

Player A

Player B		A_1	A_2	A_3	A_4	A_5	A_6
	B_1	4	2	0	2	1	1
	B_2	4	3	1	3	2	2
	B_3	4	3	7	−5	1	2
	B_4	4	3	4	−1	2	2
	B_5	4	3	3	−2	2	2

(e) Prove that any points of a convex polyhedorn can be expressed as a convex combination of its extreme points.

(f) Prove that the number of basic variables in a transportation problem with 2 origins and 3 destinations is at most 4.

3. Answer *any two* questions: 10×2=20

(a) (i) Use Vogel's Approximation Method to find the initial B.F.S. of the following transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
b_j	20	40	30	10	

(ii) Solve graphically the game whose payoff matrix is given below: 5+5

	Player B		
		B_1	B_2
Player A	A_1	2	7
	A_2	3	5
	A_3	11	2

(b) (i) Prove that the set of optimal strategies for each player in an $m \times n$ matrix game is a convex set.

(ii) Solve the travelling salesman problem: 5+5

	To					
		A	B	C	D	E
From	A	∞	6	12	6	4
	B	6	∞	10	5	4
	C	8	7	∞	11	3
	D	5	4	11	∞	5
	E	5	2	7	8	∞

(c) (i) Find the maximum value of $Z = 6x + 8y$.

subject to $5x + 2y \leq 20$

$x + 2y \geq 10$

$x, y \geq 0$

by solving its dual problem.

5+5

(ii) Solve the following assignment problem:

	A	B	C	D	E
1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

(d) (i) Solve the following LPP by two phase method:

$$\text{Maximize } z = 3x_1 - x_2$$
$$\text{subject to } \quad 2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 2$$
$$x_1 \leq 4$$
$$\text{and } \quad x_1, x_2 \geq 0$$

(ii) Is

	D_1	D_2	D_3	D_4
O_1			50	20
O_2	55			
O_3	30	35		25

an optimal solution of the following transportation problem? 5+5

	D_1	D_2	D_3	D_4	a_i
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	90
b_j	85	35	50	45	

If not, modify it to obtain a better feasible solution.