B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Physics

Course : DSE-1

(Advanced Mathematical Physics)

Time: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 2×5=10

   (a) For two matrices A and B, if $AB = BA$, show that $(AB)^n = A^n B^n$.

   (b) Prove that Anti-Hermitian matrices have pure imaginary or zero eigenvalues.

   (c) State the Cayley-Hamilton theorem for square matrix.

   (d) Find a vector parallel to xy-plane and perpendicular to $4\hat{i} - 3\hat{j} + \hat{k}$.

   (e) Give two examples of physical quantities which are Tensor of Rank 1 and Tensor of Rank 2.

   (f) Obtain the metric tensor g_{ij} for two-dimensional plane polar coordinates.

   (g) Express the components of a cross-product vector $\vec{C} = \vec{A} \times \vec{B}$ in terms of ϵ_{ijk} and the components of $\vec{A}$ and $\vec{B}$.

   (h) Expand the term of Tensor $S = a_{mn}x^m x^n$, taking $(m, n = 1, 2, 3)$

2. Answer *any two* questions: 5×2=10

   (a) Let (x, y) denote coordinates in a rectangular Cartesian coordinate systems S and (x', y') denote coordinates in coordinate systems S', related by the equations

   $$x' = 2x + 3y$$

   $$y' = -3x + 4y.$$

   Then find the area element in S' system which was $dxdy$ in S frame.

   (b) If H is a Hermitian matrix and I is a unit matrix, determine whether P is a unitary matrix or not. Here $P = (I - iH)(I + iH)^{-1}$

   (c) Prove that there is no distribution between contravariant and covariant vectors if the transformation law is of the form

   $$\bar{x}^i = a^i_m x^m + b^i$$

   Given all the a's and b's are constants such that $a^i_r a^i_m = \delta^r_m$.

   (d) Given $A_k = \frac{1}{2}\epsilon_{ijk}B^{ij}$ with B^{ij} anti-symmetric. Then show that $B^{mn} = \epsilon^{mnk}A_k$.

3. Answer *any two* questions: 10×2=20

(a) Let us consider a vector space defined by two orthogonal vectors $\vec{g} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\vec{e} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

There are two matrices σ^+ and σ^- which acts on these vectors obeying the following rules:

$\sigma^+ \vec{g} = \vec{e}, \qquad \sigma^+ \vec{e} = 0, \qquad \sigma^- \vec{e} = \vec{g}, \qquad \sigma^- \vec{g} = 0$

Find the 2×2 matrix form of these matrix-operators σ^+ and σ^-. Find the eigenvalues of a new matrix σ^z which is defined as $\sigma^z = \sigma^+\sigma^- - \sigma^-\sigma^+$. 6+4

(b) Find the eigenvalues and normalized eigenvectors of the matrix $B = \begin{pmatrix} 1 & 0 & 3 \\ 3 & 1 & 3 \\ 6 & 0 & 4 \end{pmatrix}$.

Now use the normalized eigenvectors to construct a unitary matrix and show that this matrix diagonalises the matrix B. 5+5

(c) How do you define inner product of two tensors? Let A^{pq}_{rst} be a tensor; choose $p = t$ and $q = s$ and show that A^{pq}_{rqp} is also a tensor. What is its rank? 2+6+2

(d) In Minkowski space we define $x_1 = x, x_2 = y, x_3 = z$ and $x_0 = ct$. This is done so that the space time interval can be defined as $ds^2 = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2$, here c is speed of light. Show that the metric in Minkowski space is

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

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