B.A./B.Sc. 3rd Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

(Numerical Methods)

Paper : BMH3 CC-07

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

(Notations and Symbols have their usual meaning.)

Group A

1. Answer *any five* questions: 2×5=10

(a) If $x = 3{\cdot}21$ and $y = 5{\cdot}32$ have absolute errors $\Delta x = 0{\cdot}004$ and $\Delta y = 0{\cdot}007$, find the relative error in $x + y$.

(b) If $f(x) = e^{2x}$ then prove that $f(0), \Delta f(0)$ and $\Delta^2 f(0)$ are in geometric progression.

(c) Using Lagrange's Interpolation formula, express $(3x^2 + 5x + 5)/(x^3 - 6x^2 + 11x - 6)$ as the sum of partial fractions.

(d) State conditions of convergence of Newton-Raphson method. When does the method fail?

(e) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by Gauss-elimination method.

(f) If S_1, S_2 denote the Simpson's approximations to $I = \int_a^b f(x)dx$ with 1, 2 subintervals, respectively, then show that $I = \frac{16S_2 - S_1}{15}$.

(g) Prove that $e^{hD} - 1 = \Delta$, where symbols have their usual meaning.

(h) Establish the following statement: Euler's method is a first order Runge-Kutta method.

Group B

2. Answer *any two* questions: 5×2=10

(a) (i) If α be a real root of the equation $x^3 - x^2 - 1 = 0$ then prove that the iteration formula $x_{n+1} = (\beta x_n + x_n^{-2} + 1)/(\beta + 1)$ gives the fastest convergence if $\beta = 2\alpha^{-3}$.

(ii) If $0{\cdot}667$ be an approximate value of 2/3, find the percentage error. 4+1=5

(b) Determine a, b, c such that the formula $\int_0^h f(x)dx = h[a\,f(0) + bf\left(\frac{h}{3}\right) + cf(h)]$ is exact for polynomials of as high order as possible and determine the order of the truncation error. 3+2=5

(c) Solve the following system of linear equations by Gauss-Jordan method.

$$3x_1 + 2x_2 + 3x_3 = 18$$
$$2x_1 + x_2 + x_3 = 10$$
$$x_1 + 4x_2 + 9x_3 = 16$$

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(d) Describe modified Euler's method for numerical solution of a given initial value problem. Interpret this method geometrically. 3+2=5

Group C

3. Answer *any two* questions: 10×2=20

(a) (i) Deduce Lagrange's interpolation formula from Newton's divided difference interpolation formula.

(ii) What is pivoting? Why pivoting is necessary to solve a system of equations using Gaussian elimination method?

(iii) State a sufficient condition for convergence of Gauss-Seidel method. 5+(1+2)+2=10

(b) (i) Choose the correct answer.

To find the root of $x^3 = x + 1$ in (1, 2) by iteration method, the iteration function should be written as (I) $x^3 - 1$ (II) $(x+1)^{1/3}$ (III) $(x^2-1)^{-1}$.

(ii) Show that Newton-Raphson method has a quadratic rate of convergence.

(iii) Find the largest eigenvalue in magnitude and the corresponding eigenvector of the matrix $\begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ using the power method. 3+3+4=10

(c) (i) Carry out an error estimate to approximate $y = f(x)$ whose tabular values are known at $(n+1)$ points by an interpolating polynomial.

(ii) Using Newton's forward interpolation formula obtain the expression of $f'(x)$.

(iii) Obtain $f'(1{\cdot}2)$ for the function $y = f(x)$ given in the table:

x :	0	0·4	0·8	1·2
$f(x)$:	0·000	0·493	2·022	4·666

4+3+3=10

(d) (i) Prove that $C_k^n = C_{n-k}^n$, where $C_k^n = \frac{1}{n}\int_0^n L_k\, du$ are Cote's numbers and L_k is the Lagrange's interpolation polynomial.

(ii) Given $\frac{dy}{dx} = y^2 - x^2$, where $y(0) = 2$. Find $y(0 \cdot 1)$ and $y(0 \cdot 2)$ by second order Runge-Kutta method.

(iii) Find the error propagated by the use of fourth order Runge-Kutta method.

3+(2+2)+3=10