B.A/B.Sc. 3rd Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMH3SEC11

(Logic and Sets)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning.

Group-A

1. Answer *any five* questions: 2×5=10

(a) Let p, q be statements for which the implication $p \rightarrow q$ is false. Determine the truth values for the following:

(i) $p \wedge q$

(ii) $\sim q \rightarrow \sim p$

(b) Construct a truth table for the following compound statement $\sim (p \vee \sim q) \rightarrow \sim p$, where p, q being two statements.

(c) Express in symbolic form using quantifiers $\lim\limits_{n\to\infty} x_n \neq l$, where $\{x_n\}$ is a real sequence and l is a real number.

(d) (i) Find the number of reflexive relations for a set A, where $|A| = 3$.

(ii) If A and B are two sets such that $|A| = 3$ and $|B| = 4$, then find the number of binary relations on the set $A \times B$.

(e) For any three subsets A, B and C of a universal set U, if $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then show that $B = C$.

(f) Show that if $a\,\rho\, b$ iff $7/(a-b)\,, a, b\ \in \mathbb{Z}$ then ρ is an equivalence relation.

(g) Find $P\big(P(\{0,1\})\big)$, $P(A)$ being power set of A.

(h) Give an example of a relation which is reflexive and symmetric but not transitive.

Group B

2. Answer *any two* questions: 5×2=10

(a) (i) For statements p and q verify that $p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.

(ii) Negate the following and simplify the resulting statement

$$p \wedge (q \vee r) \wedge (\sim p \vee \sim q \vee r)$$ 2+3=5

(b) Let p, q and r be three statements. Then prove the following:

(i) $p \to q \Leftrightarrow \sim p \vee q$.

(ii) $(p \to q) \wedge (r \to q) \Leftrightarrow p \vee r \to q$.

2+3=5

(c) (i) Show that for any two sets A, B $A \Delta (A \Delta B) = B$.

(ii) If $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 8\}$ and $C = \{2, 4, 6, 8, 10\}$ then find $(A \Delta B) \times C - A \times (B \Delta C)$

2+3=5

(d) (i) Show that the usual inclusion relation '$\subset$' is a P.O.R. (Partially ordered relation) on P(A), the power set of a set A.

(ii) Let $S = \{0, 1\}$. Define a relation ρ on S^3 by "$(a_1, a_2, a_3)\ \rho\ (b_1, b_2, b_3)$ iff $a_1 \leq b_1,\ a_2 \leq b_2,\ a_3 \leq b_3$" for $a_i,\ b_i \in S, i = 1, 2, 3$. Show that (S^3, ρ) is a poset.

2+3=5

Group-C

3. Answer *any two* questions: 10×2=20

(a) (i) Let the universe for the variables in the following statements consist of all real numbers. In each case negate and simplify the given statements:

(I) $\forall x\, \forall y\, [(x > y) \to (x - y > 0)]$

(II) $\forall x\, \forall y [(x < y) \to \exists z\, (x < z < y)]$

(III) $\forall x\, \forall y [(|x| = |y|) \to (y = \pm x)]$

(ii) For statements p, q, r and s verify that each of the following is a logical implication:

(I) $[(p \to q) \wedge (r \to s) \wedge (p \vee r)] \to (q \vee s)$.

(II) $[(p \to q) \wedge (r \to s) \wedge (\sim q \vee \sim s)] \to (\sim p \vee \sim r)$.

6+4=10

(b) (i) Symbolize the following:

(I) Some reals are not rationals.

(II) A professor is a good teacher if he is both well informed and well behaved.

(ii) If P_x : 'x is prime', E_x : 'x is even' and D_{xy} : 'x divides y', where the universe comprises of all integers. Then translate each of the following into English sentence:

(I) $E_2 \wedge P_2$

(II) $(\forall x)\left(E_x \Rightarrow (\forall y)(D_{xy} \Rightarrow E_y)\right)$

(III) $(\forall x)\ (D_{2x} \Rightarrow E_x)$

(IV) $(\exists x)\ (E_x \wedge D_{x6})$.

(iii) Define ρ on the set of integers $\mathbb{Z}$ by $a\ \rho\ b$ if $|a - b| \leq 3$. Examine if ρ is transitive.

2+(1+2+1½+1½)+2=10

(c) (i) If $(A \cap C) \cup (B \cap C') = \phi$, then prove that $A \cap B = \phi$, C' being complement of C.

(ii) State Fundamental theorem on equivalence relation and illustrate it by giving an example.

(iii) If $n(A) = 100$, $n(B) = 90$, $n(C) = 120$, $n(A \cap B) = 60$, $n(B \cap C) = 40$, $n(C \cap A) = 45$ *and* $n(A \cup B \cup C) = 200$, then by drawing a Venn diagram find $n(A \cap B \cap C)$, where $n(A)$ denotes the number of elements of a finite set A. 3+4+3=10

(d) (i) If $A_n = \left(2 - \frac{1}{n}, 5 + \frac{1}{n}\right]$, $B_n = \left[2 + \frac{1}{n}, 5 - \frac{1}{n}\right)$, for n = 1, 2, 3... , then find

(I) $\bigcup_{n=1}^{\infty} A_n$

(II) $\bigcap_{n=1}^{\infty} B_n$

(III) $\bigcap_{n=1}^{\infty} (A_n - B_n)$

(ii) Show that the relation R defined by " (a, b) R (c, d) iff ad = bc" is an equivalence relation on the set $\mathbb{Z} \times \mathbb{Z}$, $\mathbb{Z}$ being the set of all integers.

(iii) If R be an equivalence relation in a set A, then show that R^{-1} is also an equivalence relation in A. 4+3+3=10