B.A./B.Sc 4th Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMH4SEC 21

(Graph Theory)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning.]

Group-A

Marks : 10

1. Answer *any five* questions: 2×5=10

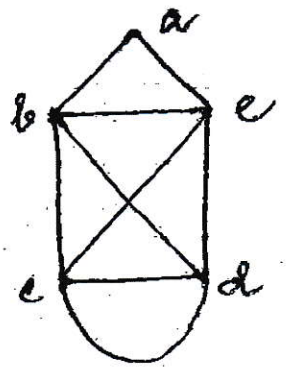
(a) Define a graph.

(b) How many vertices are there in a graph with 15 edges if each vertex is of degree 3?

(c) Define a Bipartite graph. Give an example of it. 1+1=2

(d) Define Adjacency Matrix of a graph.

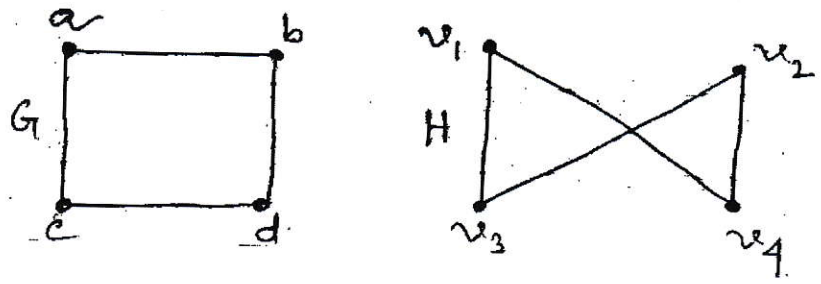
(e) Define Euler circuit. Find, if possible, an Euler circuit in the following graph. 1+1=2



(f) Define a Tree and a Binary Tree. 1+1=2

(g) Define a spanning tree with graphical representation.

(h) Examine whether the following two graphs are isomorphic or not.



Group-B

Marks : 10

2. Answer *any two* questions: 5×2=10

(a) Give an example in each of the following case:

(i) An Eulerian graph which is not Hamiltonian.

(ii) A Hamiltonian graph which is not Eulerian.

(iii) A graph which is both Eulerian and Hamiltonian.

(iv) A graph which is neither Eulerian nor Hamiltonian.

(b) Prove that every walk in a graph between two vertices u and v contains a path between u and v.

(c) Prove that a connected graph with n-vertices is a tree if and only if it has exactly $(n - 1)$ edges.

(d) Prove that a connected graph is Eulerian if and only if the degree of each vertex is even.

Group-C

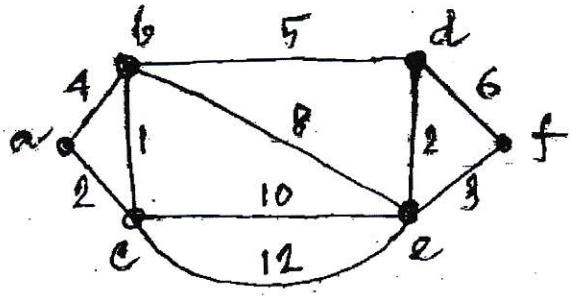
Marks : 20

3. Answer *any two* questions: 10×2=20

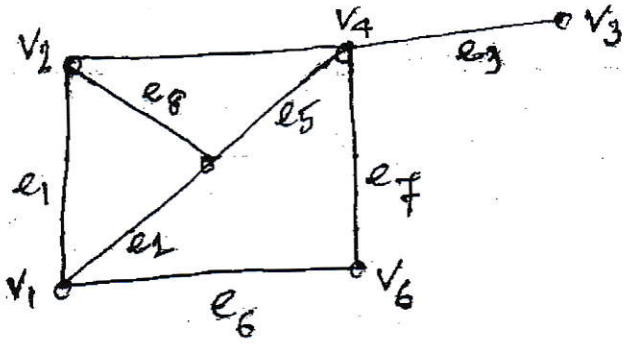
(a) (i) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-1)(n-k+1)}{2}$ edges.

(ii) Prove that the maximum number of edges in a connected simple graph with n vertices is $\frac{n(n-1)}{2}$. 6+4=10

(b) (i) Applying Dijkstra's method find the shortest path and distance between the two vertices a and f in the given following graph.



(ii) Determine the adjacency matrix of the given graph: 5+5=10



(c) Write short notes on the following:

(i) The travelling salesman problem

(ii) Königsberg Bridge Problem 5+5=10

(d) (i) Obtain a necessary and sufficient condition for a simple graph to be bipartite.

(ii) Define a minimally connected graph. Prove that a graph is minimally connected if and only if it is a tree. 5+(1+4)=10