B.A./B.Sc. 5th Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMH5CC12

(Mechanics-I)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols bear usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Write down the intrinsic equation of a common catenary explaining the symbols used.

(b) State the principle of Virtual Work for a Particle.

(c) If an inertia matrix be such that all the off-diagonal elements are zero, what can you say about the diagonal elements and the co-ordinate axes involved?

(d) Define angle of friction and cone of friction.

(e) When is a statical equilibrium said to be unstable?

(f) Find the resultant of two simple harmonic motions having slightly different periods.

(g) Prove that a central orbit is a plane curve.

(h) Prove that at an apse, the particle moves at right angle to the radius vector.

(i) Define 'Centre of Percussion' and the 'Line of Percussion'.

(j) Explain the concept of 'Momental ellipsoid'.

(k) Define areal velocity of a particle moving along a plane curve.

(l) State D'Alembert's principle.

(m) Find the moment of inertia of an elliptic plate about an axis through the centre and perpendicular to the plate.

(n) What do you mean by constraint on a dynamical system? Give an example of it.

(o) State the principle of conservation of linear momentum for a system of conservative forces.

2. Answer *any four* questions from the following: 5×4=20

(a) Show that the differential equation of the path of a particle in a plane curve under a central attractive force F is $u+\frac{d^2u}{d\theta^2}=\frac{F}{h^2u^2}$ (with usual notation). Also prove that $v^2=h^2\left[u^2+\left(\frac{du}{d\theta}\right)^2\right]$. 3+2=5

(b) Prove that in a catenary of uniform strength (i) the tension varies as the radius of curvature and (ii) the projection of the radius of curvature on the vertical is constant. 2+3=5

(c) A system of n particles is moving under external forces and their mutual actions and reactions. Write down the equation of motion of the ith particle and obtain the equation of motion of the centre of mass. 1+4=5

(d) (i) Find the centre of gravity of a uniform sector of a circle.

(ii) A uniform cubical box of edge a is placed on the top of a fixed sphere. Show that the least radius of the sphere for which the equilibrium will be stable is $\frac{a}{2}$. 3+2=5

(e) (i) State Kepler's laws of planetary motion.

(ii) A particle describes an ellipse under a force $\{\mu \div (\text{distance})^2\}$ towards a focus. If it was projected with a velocity V from a point at a distance R from the centre of force, then show that the periodic time is $\frac{2\pi}{\sqrt{\mu}}\left(\frac{2}{R}-\frac{V^2}{\mu}\right)^{-3/2}$. 2+3=5

(f) (i) Define degrees of freedom of a system of particles.

(ii) Show that the momental ellipsoid at the centre of an elliptic plate is $\frac{x^2}{a^2}+\frac{y^2}{b^2}+z^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)=$ constant. 1+4=5

3. Answer *any two* questions from the following: 10×2=20

(a) (i) Two masses M and m are connected by a string which passes through a hole in a smooth horizontal table, m hanging vertically. Show that M describes a curve whose differential equation is $\left(1+\frac{m}{M}\right)\frac{d^2u}{d\theta^2}+u=\frac{mg}{M}\frac{1}{h^2u^2}$.

(ii) Three forces P, Q, R act along the sides of the triangle formed by the lines $x+y=1$, $y-x=1$, $y=2$. Find the equation of the line of action of their resultant. Also find the magnitude of the resultant. 5+(4+1)=10

(b) (i) A force P acts along the x-axis and another force $2P$ acts along the generator of the cylinder $x^2+y^2=a^2$. Show that the Poinsot's central axis lies on the cyclinder $4(2x-z^2)+5y^2=16a^2$.

(ii) A rhombus $ABCD$ is formed of four uniform rods and suspended from the point A; it is kept in position by a light rod joining the mid-points of BC and CD. If T be the thrust on this rod and W be the weight of the rhombus, prove that $T=W\tan\frac{A}{2}$. 5+5=10

(c) (i) Show that the Kinetic Energy of a rigid body moving in two dimensions is given by $\frac{1}{2}MV^2+\frac{1}{2}MK^2\dot{\theta}^2$.

(ii) Show that $MK^2\ddot{\theta}=L$. 5+5=10

(d) (i) What is escape velocity?

(ii) A circular orbit of radius a is described under central attractive force $f(r)=\mu\left[\frac{b}{r^2}+\frac{c}{r^4}\right], \mu>0$. Prove that the motion is stable if $a^2b-c>0$.

(iii) A bead moves along a rough curved wire which is such that it changes its direction of motion with constant angular velocity. Show that a possible form of the wire is equiangular spiral. 1+4+5=10

———