B.A./B.Sc. 5th Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMH5DSE11

(Linear Programing)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Verify whether the following system of equations have degenerate/non-degenerate solution: $x_1 - 2x_3 = 0,\ x_2 + x_3 = 1, x_1, x_2, x_3 \geq 0$.

(b) Find value of p for which the three vectors $(2,\ p,\ 8)$, $(1,\ 0,\ 4)$, $(p,\ 2,\ 4)$ are linearly independent in E^3 space.

(c) Show that the set $C = \{(x_1, x_2): x_1^2 + x_2^2 \leq 1\}$ is convex set. What are the extreme points?

(d) Prove that a hyperplane and a closed half space in E_n are unbounded closed convex sets.

(e) Write down the disadvantages of Big-M method for solving L.P.P.

(f) Justify the statement — 'The solution of a transportation problem is never unbounded'.

(g) Define mixed strategies in case of a rectangular game. Under what condition mixed strategies be reduced to a pure strategy.

(h) How saddle points are defined in case of mixed strategy games?

(i) State the fundamental theorem of linear programming.

(j) Define 'Convex hull' and 'simplex'. Give examples.

(k) For what value of λ, the following game is strictly determinable:

		B	
	λ	7	3
A	–2	λ	–8
	–3	4	λ

(l) Write down the properties of a loop is a transportation problem.

(m) State the Fundamental theorem of duality.

(n) Write down the general transportation problem involving m origins and n destinations as an linear programming problem.

(o) Give the mathematical formulation of an assignment problem.

2. Answer *any four* questions from the following: 5×4=20

(a) (i) If ith constraint is equation and jth variable of the primal problem is unrestricted in sign then what will be the nature of ith variable and jth constraint of the dual problem?

(ii) What is an unbalanced assignment problem? How do you solve it? (1+1)+(1+2)=5

(b) A dealer of used car wishes to stock up his lot to maximize his profit. He can select cars A, B and C which are valued wholesale at Rs. 5000, Rs. 7000, Rs. 8000 respectively. These can be sold at Rs. 6000, Rs. 8500 and Rs. 10,500 respectively. For each type of car the probability of sale are:

Type :	A	B	C
Probability :	0·7	0·8	0·6

For every two cars of B he should buy one car of type A or C. He has Rs. 1,00,000 to invest. Formulate a linear programming problem to maximize his expected gain. 5

(c) Solve the following LPP:

Maximize $z = 2x_1 + 3x_2 + x_3$

Subject to $-3x_1 + 2x_2 + 3x_3 = 8$

$-3x_1 + 4x_2 + 2x_3 = 7$

and $x_1, x_2, x_3 \geq 0$

Comment on the type of solution you got. 5

(d) Consider the problem of assigning five operators to five machines. The assignment costs in rupees are given in the adjacent table:

	1	2	3	4	5
A	8	4	2	6	1
B	0	–	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	–	5

Operator B cannot be assigned to machine 2 and operator E cannot be assigned to machine 4. Find the optimal cost of assignment. 5

(e) If a transportation problem has m origin and n destinations, then prove that the number of basic variables in that transportation problem is at most $(m + n - 1)$. Also show that the transportation problem always has a feasible solution. 3+2=5

(f) In a two-person game, each player simultaneously shows either one or two fingers. If the number of the fingers matches, then the player A wins a rupee from the player B, otherwise A pays a rupee to B. Find the pay off matrix for this game and solve it by reducing it to a LPP. 1+1+3=5

3. Answer *any two* questions from the following: 10×2=20

(a) (i) Solve, if possible, by Dual Simplex method:

Minimize $z = 6x_1 + 11x_2$

Subject to $x_1 + x_2 \geq 11$

$2x_1 + 5x_2 \geq 40$

$x_1, x_2 \geq 0$

(ii) Use the penalty (Big M) method to solve the following L.P.P.:

Maximize $z = 2x_1 + x_2 + 3x_3$

Subject to $x_1 + x_2 + 2x_3 \leq 5$

$2x_1 + 3x_2 + 4x_3 = 12$

$x_1, x_2, x_3 \geq 0$

5+5=10

(b) (i) Prove that every extreme point of the convex set of all feasible solutions of the system $Ax = b, x \geq 0$ corresponds to a basic feasible solution.

(ii) Find an initial basic feasible solution of the following Transportation Problem using least cost method and then find an optimum solution of it.

Destinations

Origins		D_1	D_2	D_3	
	O_1	6	8	4	14
	O_2	4	9	3	12
	O_3	1	2	6	5
		6	10	15	

3+(3+4)=10

(c) (i) Using dominance property, reduce the following pay-off matrix to 2 × 2 matrix and hence solve the problem.

Player B

Player A		B_1	B_2	B_3	B_4
	A_1	4	2	3	2
	A_2	–2	4	6	4
	A_3	2	1	3	5

(ii) Solve the following 3 × 2 game problem by graphical method.

Player B

Player A		B_1	B_2
	A_1	2	3
	A_2	–2	5
	A_3	0	–1

(3+3)+4=10

(d) (i) Is

	1	2	3	4
1			50	20
2	55			
3	30	35		25

an optimal solution of the following Transportation problem?

	1	2	3	4	a_i
1	6	1	9	3	70
2	11	5	2	8	55
3	10	12	4	7	90
b_j	85	35	50	45	

If not, find the optimal solution.

(ii) Solve the travelling salesman problem given by the following 5×5 matrices:

		TO				
		1	2	3	4	5
	1	∞	14	10	24	41
FROM	2	6	∞	10	12	10
	3	7	13	∞	8	15
	4	11	14	30	∞	17
	5	6	8	12	16	∞

(1+4)+5=10
