B.Sc. 3rd Semester (Honours) Examination, 2019 (CBCS)

Subject : Physics

Paper : CC-V

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* of the following questions: 2×5=10

(a) State that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, where symbols have their usual meaning.

(b) Prove that $\frac{d}{dx}[x^n J_n(ax)] = ax^n J_{n-1}(ax)$.

(c) State the Dirichlet conditions in the context of Fourier series expansion.

(d) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(e) If $erf(x)$ be the error function of x, show that

$$\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2}[erf(b) - erf(a)].$$

(f) Express electrostatic potential between two electric charges at a distance d apart, as a series of Legendre polynomial.

(g) What type of partial differential equations are the following ones?

(i) $\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0$

(ii) $\frac{\partial^2 u(x,y)}{\partial x^2} - k^2 \frac{\partial u(x,y)}{\partial y} = 0$, k is a constant.

(h) What is significant digit?

Find the loss of significant digits by subtracting 0·75288 from 0·75289.

2. Answer *any two* of the following questions: 5×2=10

(a) Find the Fourier series of the function

$$f(x) = \begin{cases} -k \text{ for } -\pi < x < 0 \\ +k \text{ for } 0 < x < \pi \end{cases} \text{ and } f(x+2\pi) = f(x).$$

(b) Starting from Beta function, prove that

(i) $\Gamma(2m) = 2^{2m-1}(\pi)^{-\frac{1}{2}}\Gamma(m)\,\Gamma\left(m+\frac{1}{2}\right)$ and hence

(ii) $\left(m+\frac{1}{2}\right)! = \pi^{\frac{1}{2}}\frac{(2m+1)!!}{2^{m+1}}$, where $(2m+1)!! = 1{\cdot}3{\cdot}5 \ldots\ldots (2m-1)(2m+1)$. 3+2=5

(c) If $f(x) = 0, \quad -1 < x < 0$

$\qquad\qquad\quad 1, \quad 0 < x < 1$

show that $f(x)$ can be expanded as $\frac{1}{2}P_0(x) + \frac{3}{4}P_1(x) - \frac{7}{16}P_3(x) + \cdots$.

symbols have their usual meaning.

(d) Define absolute error and relative error. The length and breadth of a paper sheet is 19·27 cm and 13·73 cm respectively and probable errors in measuring them are 0·01 and 0·02 respectively. Find the relative error of area calculated from these data. 2+3=5

3. Answer *any two* of the following: 10×2=20

(a) (i) Write down Bessel's differential equation and determine the singular points of this differential equation.

(ii) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

(iii) Prove that $J_n''(x) = \frac{1}{4}\left[J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)\right]$. 2+4+4=10

(b) Write down complex form of Fourier series. Find complex Fourier series of $f(x) = e^{ax}$ in $-l < x < l$, where the series contains both sinh and cosh terms. 3+7=10

(c) (i) Using Beta function, evaluate $\int_0^{\frac{\pi}{2}} (\cos\theta)^r d\theta$.

(ii) Prove that

$$\int_{-1}^{+1} P_m(x)P_n(x)\, dx = 0 \text{ for } [m \neq n]$$
$$= \frac{2}{2n+1} \text{ for } [m = n].$$
4+6=10

(d) (i) A uniform flexible string of length L and mass per unit length ρ is stretched between its two ends with a tension T. Find the equation of motion of the string when it is plucked at any point and then released. What will be the suitable solution for the wave equation?

(ii) A tight straight string with fixed end points at $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set to vibration by giving to each of its points a velocity $\mu x(l - x)$, find the displacement of the string at any distance x at any time t. 3+7=10

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