B.Sc. 5th Semester (Honours) Examination, 2019 (CBCS)

Subject : Physics

Paper : CC-XI

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* of the following questions: 2×5=10

(a) What is a well-behaved wave function? Which one of $\Psi_1 = Ae^{+x^2}$ and $\Psi_2 = Ae^{-x^2}$ is a well-behaved quantum mechanical function in the range $-\alpha \leq x \leq \alpha$? Explain.

(b) Write down the time-dependent Schrödinger wave equation for a free particle in the momentum space. Obtain the solution of the wave equation.

(c) Find the value of the constant A that makes $e^{-\alpha x^2}$ an eigenfunction of the operator $\left(\frac{d^2}{dx^2} - Ax^2\right)$. What is the corresponding eigenvalue?

(d) Find the expectation value of the momentum of a particle whose normalised wave function is $\Psi(x) = Ne^{-(x^2/2a^2)+ikx}$.

(e) A one-dimensional wave function is given by $\Psi(x) = \sqrt{a}\, e^{-ax}$. Find the probability of finding the particle between $x = \frac{1}{a}$ and $x = \frac{2}{a}$.

(f) Find the value of Lande g-factor (g) for energy level $^3P_1$.

(g) Using the vector model, determine the possible values of the total angular momentum of an f-electron.

(h) What is Bohr magneton? Calculate its value.

2. Answer *any two* of the following questions: 5×2=10

(a) The Gaussian function is represented by $\Psi(x) = \frac{1}{(\sigma\sqrt{\pi})^{\frac{1}{2}}} \exp\left[-\frac{x^2}{2\sigma^2}\right]$

where σ is the width of wave packet. Calculate the momentum function $A(K)$ using Fourier Transform.

$\left[\text{Given} : \int_{-\alpha}^{+\alpha} \exp\left\{\frac{-(x+ik\sigma^2)^2}{2\sigma^2}\right\} dx = \sigma\sqrt{2\pi}\right]$

(b) Wavelengths can be determined with accuracies of one part in 10^4. What is the uncertainty in the position of 1Å X-ray photon when its wavelength is simultaneously measured?

(c) What is Zeeman effect? Describe the experimental arrangement for studying the Zeeman effect. 1+4=5

(d) (i) Establish the relation $[\hat{L}_x, \hat{L}_y] = i\hbar\, \hat{L}_z$.

(ii) Prove that the operator $\hat{p}_x$ is hermitian (symbols have their usual meaning). 3+2=5

3. Answer *any two* of the following questions: 10×2=20

(a) Write down the time-independent Schrödinger equation for the motion of the electron in hydrogen atom, assuming that the proton is at rest.

Given : $\vec{\nabla}^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$.

Separate the Schrödinger equation into one radial and two angular parts. 1+(3+3+3)=10

(b) (i) Consider a particle of mass m and energy $E < V_0$ moving in a one-dimensional square-well potential of width L and depth V_0 given by

$$\begin{aligned} V(x) &= V_0 \; at \; x < -\frac{L}{2} \\ &= 0 \; at \; -\frac{L}{2} < x < \frac{L}{2} \\ &= V_0 \; at \; x > \frac{L}{2} \end{aligned}$$

Write down the Schrödinger equations for three regions. What are the boundary conditions? Imposing boundary conditions find the well-behaved solutions in three regions. [Values of constants appearing in the solutions need not be evaluated.]

(ii) The ground state and the first excited state wave functions of an atom are Ψ_0 and Ψ_1, respectively, the corresponding energies being E_0 and E_1. If the system has a 40% probability of being found in the ground state and 60% probability of being found in the first excited state, what is the wave function of the atom and what is the average energy of the atom? (2+2+2)+(2+2)=10

(c) (i) Write down the Schrödinger equation of a one-dimensional harmonic oscillator. What is the energy of this oscillator when it is in the eigenstate associated with the quantum number n? What values may n have? What is the zero-point energy (E_0) of the oscillator?

(ii) The lowest-energy eigenfunction of the linear harmonic oscillator is

$$\Psi_0(x) = \left(\frac{2mv}{\hbar}\right)^{1/4} \exp[(-\pi m v/\hbar)x^2]$$

where m is the mass and v is the classical frequency of the oscillator.

Show that the expectation value of the potential energy $\langle V \rangle$ of the oscillator in the above state is given by $\langle V \rangle = E_0/2$. (1+1+1+1)+6=10

(d) (i) Using vector atom model determine the possible terms corresponding to the principal quantum number $n = 3$ and compute the angle between $\vec{l}$ and $\vec{s}$ vectors for the term $^2D_{5/2}$.

(ii) Consider 2 electrons : $l_1 = 3,\ s_1 = \frac{1}{2}$; $l_2 = 1,\ s_2 = \frac{1}{2}$. Find the J values assuming $J - J$ coupling.

(3+3)+4=10

Useful Data :

$h = 6{\cdot}626 \times 10^{-34}$ Joule – Sec

$m_e = 9{\cdot}108 \times 10^{-31}$ Kg

$e = 1{\cdot}602 \times 10^{-19}$ Coulomb

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