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I wish him all success in life.

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I wish him all success in life.

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Siliguri

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RESEARCH ARTICLE

WILEY

Dynamic response of pulsatile flow of blood in a stenosed tapered artery

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The aim of this paper is to throw some light on the rheological study of pulsatile blood flow in a stenosed tapered arterial segment. Arterial wall is considered to be rigid and flexible separately for improving the similarity to the in vivo situation. The streaming blood is considered to be Newtonian. The governing nonlinear equations of motion are sought using the well-known stream function-vorticity method and are solved numerically by finite difference technique. Important rheological parameters, such as axial velocity component, wall shear stress, and flow separation region are estimated in the neighborhood of the stenosis. Effects of stenosis height, vessel tapering, and wall flexibility on the blood flow are investigated properly and are explained in detail through their graphical representations.

KEYWORDS

finite difference technique, flow separation, pulsatile flow, stenosis, stream function-vorticity method, wall shear stress

1 | INTRODUCTION

Now a days, cardiovascular diseases are one of the major causes of human mortality worldwide. One of such diseases is atherosclerosis or arterial stenosis, which is formed because of the deposition of fats and fibrous tissues on the arterial wall. Although the exact reasons for the initiation of the disease are not clearly known, it has been established that once a mild stenosis is formed, it further influences the propagation of the disease. The blood flow characteristics are altered significantly because of the formation of a stenosis.[1,2]

Locally irregular flow rate, variation of wall shear stress, and the flow separation region formed near the arterial wall help further in the development of the disease.[3,4] The changes of pattern of blood flow from the usual state to the abnormal state due to formation of stenosis are believed to be the main cause of cardiac arrest. Because of the formation of stenosis, coronary arteries become narrow. As a result, they are unable to transport sufficient amount of blood to the heart muscle for efficient functioning of heart.[5] Reduction in flow of blood due to formation of stenosis also causes debilitation. This eventually results in a cardiac arrest. Sometimes, the plaque forming a stenosis may be ruptured into particles, known as emboli, which may lodge in an artery downstream.[6] If the broken particles are carried into the brain, it causes neurological disorder or a stroke. Plaque rupture may sometimes form a thrombus that blocks blood flow to the heart causing unstable angina or myocardial infarction.[7] So blood flow becomes very complicated in nature because of the formation of stenosis, and the complexity increases with the severity of the disease. It is believed that the hemodynamic factors play an important role in the progression of the disease. For early detection of the disease and its prevention purpose, a major research work is going on worldwide.

Blood behaves like a Newtonian fluid when it flows through larger arteries at high shear rates,[8] whereas it behaves like a non-Newtonian fluid when it flows through smaller arteries[9] at low shear rates $\left(\dot{\gamma}<10\ \text{sec}^{-1}\right)$. During the past few

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Numerical simulation of physiologically relevant pulsatile flow of blood with shear-rate-dependent viscosity in a stenosed blood vessel

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Pulsatile flow of blood in a blood vessel having time-dependent shape (diameter) is investigated numerically in order to understand some important physiological phenomena in arteries. A smooth axi-symmetric cosine shaped constriction is considered. To mimic the realistic situation as far as possible, viscosity of blood is taken to be non-uniform, a shear-thinning viscosity model is considered and a physiologically relevant pulsatile flow is introduced. Taking advantage of axi-symmetry in the proposed problem, the stream function–vorticity formulation is used to solve the governing equations for blood flow. Effect of different parameters associated with the problem on the flow pattern has been investigated and disparities from the Newtonian case are discussed in detail.

Keywords: Pulsatile flow; stenosis; stream function–vorticity method; finite difference technique; non-Newtonian fluid; fluid–structure interaction; wall shear stress; flow separation.

Mathematics Subject Classification 2010: 76Z05, 92C35, 92C50

1. Introduction

In the recent past, fluid mechanical studies of blood flow through diseased arteries have drawn attention of researchers due to the several implications on human health. The partial occlusion of an artery due to the formation of an atherosclerotic lesion beneath the intima of the arterial wall by the deposition of cholesterol, calcium, some low-density lipoproteins (LDL), etc., known as stenosis, is one of the most

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RESEARCH ARTICLE

WILEY

Effects of variable viscosity on pulsatile flow of blood in a tapered stenotic flexible artery

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The objective of the present study is to investigate the effects of variable viscosity on incompressible laminar pulsatile flow of blood through an overlapping doubly constricted tapered artery. To mimic the realistic situation, wall of the artery is taken to be flexible, and physiologically relevant pulsatile flow is introduced. The governing equations of blood flow are made dimensionless. A coordinate transformation is used to make the overlapping doubly constricted wall geometry of tube to a straight tube. Taking advantage of the Stream function–Vorticity formulation, the system of partial differential equations is then solved numerically by finite difference approximations. Effects of Reynolds number, Strouhal number, degree of contraction, tapering angle, and viscosity parameters are presented graphically and analyzed. The results show that formation of stenosis and tapering disturb the flow field significantly, and degree of stenosis is more important in influencing blood flow compared with tapering.

KEYWORDS
finite difference, flow separation, overlapping stenoses, pulsatile flow, tapering angle, variable blood viscosity

1 | INTRODUCTION

Nowadays, cardiovascular diseases have been detected as one of the major sicknesses by which several people suffer. Atherosclerosis or arterial stenosis is a common cardiovascular disease, which frequently occurs in human being in developed nations. Formation of stenosis due to deposition of lipids, low density lipoproteins, and triglycerides on the inner surface of arterial wall reduces the cross-sectional area of arterial lumen and changes the regional blood rheology thereby, which, in turn, severely affects human health. Atherosclerotic lesions are usually formed and grow at certain locations of the arterial tree, such as at curving, branching, and bifurcation sites. Although the exact causes for the initiation of a stenosis are not clearly known, it has been found that the proliferation of the disease is closely related to local hemodynamic factors, such as wall shear stress distribution (Smedby,[1] Liepsch[2]).

Over the past few decades, blood flow through stenotic arteries has drawn significant attention to the researchers and become an interesting research area to work on. Several researchers modeled steady flow of blood through stenosed rigid artery by considering the flowing blood as a Newtonian fluid (Lee,[3] Deshpande et al[4]). Because of the periodic pumping action of the heart, blood flow in arteries is pulsatile in nature. Treating blood as Newtonian fluid, Mustapha et al,[5] Liu et al,[6] Paul and Molla,[7] Mandal et al[8] investigated unsteady flow of blood through stenotic rigid artery with pulsation. Goswami et al[9] compared the simple and physiological pulsatile flow behaviors of blood in a stenosed artery. Considering blood as a non-Newtonian fluid, Mandal et al[10] investigated the pulsatile flow in a stenosed rigid artery.

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Numerical Simulation of Mass Transfer in Pulsatile Flow of Blood Characterized by Carreau Model under Stenotic Condition

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ABSTRACT

The present numerical study deals with a mathematical model representing mass transfer in blood flow under stenotic condition. Streaming blood is considered as a non-Newtonian fluid characterized by Carreau fluid model and the vessel wall is taken to be flexible. The nonlinear pulsatile flow phenomenon is governed by the Navier-Stokes equations together with the continuity equation while that of mass transfer is governed by the convection-diffusion equation coupled with the velocity field. A finite difference scheme is developed to solve these equations accompanied bysuitable initial and boundary conditions. Results obtained are examined for numerical stability up to wanted degree of correctness. Various significant hemodynamic parameters are examined for additional qualitative insight of the flow-field and concentration-field over the entire arterial segment with the help of the obtained numerical results. Comparisons are made with the available results in open literature and good agreement has been achieved between these two results. Comparisons have been made to understand the effects of viscosity models for Newtonian and non-Newtonian fluids and also for rigid and flexible arteries.

Keywords: Non-Newtonian fluid; Carreau fluid model; Pulsatile flow; Mass transfer; Flexible artery.

1. INTRODUCTION

Partial occlusion of arteries, known as arterial stenosis, is one of the most frequent anomalies in cardiovascular system. Due to accumulation of low-density lipoprotein and other lipid bearing materials in streaming blood, such type of constrictions are formed (Ross 1993) and the disease thus caused by is called atherosclerosis. Under physiological conditions, atherosclerotic plaques may burst with no notice and as a result heart attack and stroke occur (Haque *et al*. 2014). Though the accurate grounds behind thecommencement of such constriction are not yet clearly known but it is well recognized that once such constriction is shaped, the hemodynamic environment in the area of the constriction is drastically changed and fluid dynamic factors take part in the propagation of the disease (Friedman *et al*. 1992; Smedby 1997; Liepsch 2002). Such obstruction in arteries implies that the transport of low-density lipoproteins from blood stream onto the arterial wall must play a key role in the development of stenotic lesions. Moreover, mechanical stresses are created by the interactions of plaque with the flow of blood leading to its burst. Recirculation region is formed downstream the plaque (Haque *et al*. 2014).

The flow disturbances associated with a medium degree of stenosis can be detected through the use of non-invasive methods such as the Doppler ultra-sound technique, but a method to detect a mild stenosis is still out of hand. The ability to describe the flow through constricted arteries may provide the possibility of diagnosing the disease in its earlier stages, even before the stenosis become clinically relevant, and is the basis for surgical intervention. Computational fluid dynamics provides a useful and non-invasive tool to study the hemodynamic factors, suspected to be associated with the propagation of atherosclerosis, through stenosed arteries (Pontrelli 2001).

During the past few decades, several studies on fluid dynamics through constricted arteries have been carried out to evaluate the flow pattern and the wall shear stress under steady and pulsatile flow

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Heat transfer in pulsatile blood flow obeying Cross viscosity model through an artery with aneurysm

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Abstract A mathematical model has been developed to represent heat transfer in pulsatile blood flow through an artery having an aneurysm in its lumen occurring from different types of anomalous enlargement. Time-variant wall geometry has been considered and the streaming blood is taken as a non-Newtonian fluid obeying Cross viscosity model. With the help of stream function–vorticity method, a finite difference scheme is used to solve the governing equations along with the suitable initial and boundary conditions in order to find out the physiologically noteworthy parameters up to the required degree of precision. Particular importance has accordingly been paid in comparing the current numerical results with the existing ones, and an excellent conformity between these two has been attained. For additional qualitative insight into the flow and heat transfer, effects of severity of aneurysm and different hemodynamic parameters on axial velocity, wall shear stress, and heat transfer rate are presented through graphical representations and analyzed in detail.

Keywords Aneurysm · Blood flow · Cross viscosity model · Heat transfer · Stream function–vorticity method

1 Introduction

From the theoretical, experimental, and clinical perspectives, blood flow through a diseased artery is an attractive field of research. Unusual expansion in the arterial wall is supposed by many researchers to play a major role in the formation of a disease, a leading cause of mortality in the present world. Several mathematical models have been constructed to explain the capacity to understand the accessible treatment. Weakening of vessel wall in certain locations forms a blood-filled balloon-like dilatation known as an aneurysm. Normal flow of blood through arteries thus becomes perturbed and complicated. Rhythmic flow of blood inside an aneurysm worsens the structure, and rupture may happen ultimately. The most common sites of aneurysm are the arteries such as cerebral, carotid,

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ORIGINAL ARTICLE

Check for updates

Pulsatile flow of blood with shear-dependent viscosity through a flexible stenosed artery in the presence of body acceleration

Subrata Mukhopadhyay[1] · Mani Shankar Mandal[2] · Swati Mukhopadhyay[3]

Received: 3 December 2020 / Revised: 20 November 2021 / Accepted: 17 January 2022

Abstract

A mathematical model of physiological pulsatile flow of blood through a stenotic flexible artery in the presence of body acceleration is presented in this paper. Streaming blood is considered as a shear-thinning non-Newtonian fluid as proposed by Yeleswarapu (Evaluation of continuum models for characterizing the constitutive behaviour of blood, Ph.D. thesis, Dept. Mech. Eng., University of Pittsburgh, 1996), and a physiological pulsatile flow rate proposed by Pedrizzetti (J Fluid Mech 310:89–111, 1996) has been taken through the tube. Deformation of vessel wall is modelled as a function of flow rate. This computational study of an idealized model may bring some insights for realistic blood flow through a stenotic artery. The novelty of this work lies in the fact that realistic flow of blood through a stenosed artery has been studied as far as possible and a new idea has been provided to describe the arterial wall motion. Governing equations in cylindrical polar coordinates are solved using stream function–vorticity method. Behaviour of various flow quantities is investigated through a parametric study. It is noted that the degree of constriction and body acceleration have important impacts on the haemodynamic parameters such as wall shear stress, oscillatory shear index, and relative residence time. Increasing body acceleration enhances the peak value of wall shear stress, but reduces the oscillatory shear index and relative residence time. Almost 1/4th increase in length of flow separation is found when Froude number raises its value from 0.1 to 0.5, other parametric values remaining fixed. On the other hand, almost 50% increase in the magnitude of the peak value of wall pressure is found when the amplitude of body acceleration takes a value 0.4 ($A = 0.4$) compared to the without body acceleration case ($A = 0$). These results have a significant role.

Keywords Non-Newtonian fluid · Pulsatile flow · Stenotic artery · Body acceleration · Finite difference

1 Introduction

Studies related to flow of blood through stenotic arteries have drawn noteworthy attention to researchers during the last few decades, as the commencement and progress of many cardiovascular diseases which direct to the malfunction of the cardiovascular system are intimately associated with the haemodynamics of such an artery. Atherosclerosis or arterial stenosis commences through alteration in endothelial cell utility that encourages white blood cells to attach to the endothelium instead of flowing through the blood usually and the endothelium turns out to be damaged thereby. This permits blood cells and deadly substances, such as lipids, low density lipoproteins and triglycerides, present in the blood to go by the endothelium and mount up in this area. Different compound phenomena take place in the course of time and lastly calcium builds up over the wound site to shape a material similar to bone.

The formation of arterial stenosis reduces the supply of blood to the distal bed through that artery. With the spread of atherosclerotic plaque, if it becomes unstable, it ruptures and exposes its contents to streaming blood. Platelets may then build up around the ruptured plaque and result in blood coagulation which occludes the artery. When blood flowing through an artery is severely compromised by a blood clot, the cells of the tissues that depend on the blood flow from that artery become injured or die. Coronary atherosclerosis

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ORIGINAL ARTICLE

HEAT TRANSFER WILEY

Heat transport in magnetohydrodynamic physiological blood flow through a constricted artery

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Abstract

In this paper, we study how the magnetohydrodynamic (MHD) pulsatile flow of blood and heat transfer works through a constricted artery with a flexible wall. The human circulatory network consists of veins and arteries that sometimes contain constrictions, allowing the impact of the applied magnetic field on flow fields to be observed. The walls of the flowing medium are considered to be a function of time. The flowing blood is hypothesized as shear-thinning fluid, emulating Yeleswarapu's viscosity replica. Additionally, we consider the energy equation to understand the impact of a magnetic field on heat transfer rates for such flows. The vorticity transport equation along with the stream function equation is obtained using the vorticity–stream function technique. Numerical solutions of the governing nonlinear MHD equations and energy equation in addition to physically pertinent flow conditions were achieved by adapting a finite difference scheme. Considerable attention has been paid to ensure an accurate comparison between the current and previous results. The two sets of numbers appear to match closely. For an even deeper understanding of the flow and heat transport process, the effects of height of stenosis and diverse physiological parameters on time-averaged wall shear stress (TAWSS), rate of heat transport, and so on are explored in depth through their graphical depiction. In the vicinity of the constriction, it is observed that the

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FLUID MECHANICS

EFFECTS OF MAGNETIC FIELD ON HEAT AND MASS TRANSFER IN PULSATILE FLOW OF BLOOD UNDER STENOTIC CONDITION

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ABSTRACT: A mathematical model is developed in the present study to investigate the heat and mass transfer phenomena in blood flow under stenotic condition. The non-Newtonian Carreau fluid model is used to characterize the streaming blood. The nonlinear governing equations are solved numerically by employing a finite difference scheme along with suitable initial and boundary conditions under the action of applied magnetic field. Various significant hemodynamic parameters are examined for additional qualitative insight of the flow-field, temperature-field and concentration-field over the entire flow regime with the help of the numerical results obtained in this study. Comparisons are made with available results in open literature and are found in good agreement between these two results.

KEY WORDS: Heat and mass transfer, Non-Newtonian fluid, Magnetic field, Finite-difference.

1 INTRODUCTION

Heat and mass transfer phenomena in constricted channels have significant practical implications in engineering fields and Biorheology. The motion of an electrically conducting fluid across a magnetic field induces current, which affects the fluid flow. In fact the propagating field is influenced by Lorentz force. Understanding the physics of magnetohydrodynamic (MHD) flow, prediction of flow separation region

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