



Government of West Bengal  
Government General Degree College, Kalna-I

*Office of the Principal*

Muragacha, Medgachi, Purba Bardhaman-713405

Website: [www.ggdck.ac.in](http://www.ggdck.ac.in); E-mail: [govt.collegekalna1@gmail.com](mailto:govt.collegekalna1@gmail.com)

Date: 23-03-2024

**To Whom It May Concern**

This is to certify that the college has no objection in permitting **Dr. Mani Shankar Mandal, Assistant Professor in Mathematics** to continue his research/academic linkages/activities with the **Department of Mathematics, The University of Burdwan, Golapbug, Burdwan, West Bengal** started in **2016** without hampering the normal duties of the college.

I wish him all success in life.

Principal

( Government General Degree College, Kalna-I

**Principal**  
**Government General Degree College**  
Kalna-I

***Dr. Swati Mukhopadhyay, BOYSCAST Fellow***

**Professor**

**Department of Mathematics**

**The University of Burdwan**

Burdwan-713104, W. B., India

Mob. No.: 9735119297

E-mail: [swati\\_bumath@yahoo.co.in](mailto:swati_bumath@yahoo.co.in),

[smukhopadhyay@math.buruniv.ac.in](mailto:smukhopadhyay@math.buruniv.ac.in)



Phone : (Administrative)03422533913,  
2533914, 2533917, 2533918, 2533919  
(Academic): 0342-2556549, 2556566  
(Extn : 427)  
Fax : (91) 0342-2530452  
Direct : (91) 0342-2657741

Website : <http://buweb.buruniv.ac.in>

*Date : 11. 03. 2024*

**TO WHOM IT MAY CONCERN**

This is to certify that **Dr. Mani Shankar Mandal, Assistant Professor in Mathematics, Govt. General Degree College, Kalna-I, Purba Bardhaman-713405, West Bengal** is my long-term research collaborator. It is a great pleasure to extend my appreciation for the continued collaboration with Dr. Mandal.

Our collaborative efforts have resulted in several successful publications of research articles that have made substantial contribution in the field of Computational Fluid Dynamics. Hope our collaboration will remain as dynamic as ever.

Signature: *Swati Mukhopadhyay* 11.03. 2024


Name: SWATI MUKHOPADHYAY

Designation: PROFESSOR

Address: DEPARTMENT OF MATHEMATICS  
THE UNIVERSITY OF BURDWAN, BURDWAN-713104  
WEST BENGAL, INDIA

**Dr. Swati Mukhopadhyay**  
**Professor**  
**Department of Mathematics**  
**The University of Burdwan**  
**Burdwan-713104**

# Dynamic response of pulsatile flow of blood in a stenosed tapered artery

Subrata Mukhopadhyay<sup>1</sup> | Mani Shankar Mandal<sup>2</sup> | Swati Mukhopadhyay<sup>3</sup> 

<sup>1</sup>Department of Mathematics, Acharya Prafulla Chandra Roy Government College, Siliguri, West Bengal, India

<sup>2</sup>Department of Mathematics, Government General Degree College Kalna-1, Burdwan, West Bengal, India

<sup>3</sup>Department of Mathematics, University of Burdwan, Burdwan, West Bengal 713104, India

## Correspondence

Swati Mukhopadhyay, Department of Mathematics, University of Burdwan, Burdwan-713104, West Bengal, India.  
Email: swati\_bumath@yahoo.co.in

## Funding information

CSIR, India

Communicated by: E. Venturino

The aim of this paper is to throw some light on the rheological study of pulsatile blood flow in a stenosed tapered arterial segment. Arterial wall is considered to be rigid and flexible separately for improving the similarity to the in vivo situation. The streaming blood is considered to be Newtonian. The governing nonlinear equations of motion are sought using the well-known stream function-vorticity method and are solved numerically by finite difference technique. Important rheological parameters, such as axial velocity component, wall shear stress, and flow separation region are estimated in the neighborhood of the stenosis. Effects of stenosis height, vessel tapering, and wall flexibility on the blood flow are investigated properly and are explained in detail through their graphical representations.

## KEYWORDS

finite difference technique, flow separation, pulsatile flow, stenosis, stream function-vorticity method, wall shear stress

## 1 | INTRODUCTION

Now a days, cardiovascular diseases are one of the major causes of human mortality worldwide. One of such diseases is atherosclerosis or arterial stenosis, which is formed because of the deposition of fats and fibrous tissues on the arterial wall. Although the exact reasons for the initiation of the disease are not clearly known, it has been established that once a mild stenosis is formed, it further influences the propagation of the disease. The blood flow characteristics are altered significantly because of the formation of a stenosis.<sup>1,2</sup>

Locally irregular flow rate, variation of wall shear stress, and the flow separation region formed near the arterial wall help further in the development of the disease.<sup>3,4</sup> The changes of pattern of blood flow from the usual state to the abnormal state due to formation of stenosis are believed to be the main cause of cardiac arrest. Because of the formation of stenosis, coronary arteries become narrow. As a result, they are unable to transport sufficient amount of blood to the heart muscle for efficient functioning of heart.<sup>5</sup> Reduction in flow of blood due to formation of stenosis also causes debilitation. This eventually results in a cardiac arrest. Sometimes, the plaque forming a stenosis may be ruptured into particles, known as emboli, which may lodge in an artery downstream.<sup>6</sup> If the broken particles are carried into the brain, it causes neurological disorder or a stroke. Plaque rupture may sometimes form a thrombus that blocks blood flow to the heart causing unstable angina or myocardial infarction.<sup>7</sup> So blood flow becomes very complicated in nature because of the formation of stenosis, and the complexity increases with the severity of the disease. It is believed that the hemodynamic factors play an important role in the progression of the disease. For early detection of the disease and its prevention purpose, a major research work is going on worldwide.

Blood behaves like a Newtonian fluid when it flows through larger arteries at high shear rates,<sup>8</sup> whereas it behaves like a non-Newtonian fluid when it flows through smaller arteries<sup>9</sup> at low shear rates ( $\dot{\gamma} < 10 \text{ sec}^{-1}$ ). During the past few

## Numerical simulation of physiologically relevant pulsatile flow of blood with shear-rate-dependent viscosity in a stenosed blood vessel

Subrata Mukhopadhyay\*, Mani Shankar Mandal<sup>†</sup>  
 and Swati Mukhopadhyay<sup>‡,§</sup>

*\*Department of Mathematics  
 Acharya P. C. Roy Govt. College, Siliguri, W.B., India*

*†Department of Mathematics  
 G.G.D.C at Kalna-I, Purba Bardhaman-713405, W.B., India*

*‡Department of Mathematics  
 The University of Burdwan, Burdwan-713104, W.B., India  
 §swati\_bumath@yahoo.co.in*

Received 3 February 2018  
 Revised 1 July 2018  
 Accepted 18 July 2018  
 Published 23 August 2018

Pulsatile flow of blood in a blood vessel having time-dependent shape (diameter) is investigated numerically in order to understand some important physiological phenomena in arteries. A smooth axi-symmetric cosine shaped constriction is considered. To mimic the realistic situation as far as possible, viscosity of blood is taken to be non-uniform, a shear-thinning viscosity model is considered and a physiologically relevant pulsatile flow is introduced. Taking advantage of axi-symmetry in the proposed problem, the stream function–vorticity formulation is used to solve the governing equations for blood flow. Effect of different parameters associated with the problem on the flow pattern has been investigated and disparities from the Newtonian case are discussed in detail.

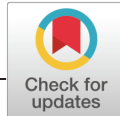
**Keywords:** Pulsatile flow; stenosis; stream function–vorticity method; finite difference technique; non-Newtonian fluid; fluid–structure interaction; wall shear stress; flow separation.

Mathematics Subject Classification 2010: 76Z05, 92C35, 92C50

### 1. Introduction

In the recent past, fluid mechanical studies of blood flow through diseased arteries have drawn attention of researchers due to the several implications on human health. The partial occlusion of an artery due to the formation of an atherosclerotic lesion beneath the intima of the arterial wall by the deposition of cholesterol, calcium, some low-density lipoproteins (LDL), etc., known as stenosis, is one of the most

<sup>§</sup>Corresponding author.



# Effects of variable viscosity on pulsatile flow of blood in a tapered stenotic flexible artery

Subrata Mukhopadhyay<sup>1</sup> | Mani Shankar Mandal<sup>2</sup> | Swati Mukhopadhyay<sup>3</sup>

<sup>1</sup>Department of Mathematics, Acharya P. C. Roy Govt. College, Siliguri, India

<sup>2</sup>Department of Mathematics, G.G.D.C at Kalna-I, Purba Bardhaman, India

<sup>3</sup>Department of Mathematics, The University of Burdwan, Burdwan, India

## Correspondence

Swati Mukhopadhyay, Department of Mathematics, The University of Burdwan, Burdwan-713104, West Bengal, India.  
Email: swati\_bumath@yahoo.co.in

## Funding information

Council of Scientific and Industrial Research, Grant/Award Number: 25(0244)/15/EMR-II Dated 07. 07. 2015

The objective of the present study is to investigate the effects of variable viscosity on incompressible laminar pulsatile flow of blood through an overlapping doubly constricted tapered artery. To mimic the realistic situation, wall of the artery is taken to be flexible, and physiologically relevant pulsatile flow is introduced. The governing equations of blood flow are made dimensionless. A coordinate transformation is used to make the overlapping doubly constricted wall geometry of tube to a straight tube. Taking advantage of the Stream function–Vorticity formulation, the system of partial differential equations is then solved numerically by finite difference approximations. Effects of Reynolds number, Strouhal number, degree of contraction, tapering angle, and viscosity parameters are presented graphically and analyzed. The results show that formation of stenosis and tapering disturb the flow field significantly, and degree of stenosis is more important in influencing blood flow compared with tapering.

## KEYWORDS

finite difference, flow separation, overlapping stenoses, pulsatile flow, tapering angle, variable blood viscosity

## 1 | INTRODUCTION

Nowadays, cardiovascular diseases have been detected as one of the major sicknesses by which several people suffer. Atherosclerosis or arterial stenosis is a common cardiovascular disease, which frequently occurs in human being in developed nations. Formation of stenosis due to deposition of lipids, low density lipoproteins, and triglycerides on the inner surface of arterial wall reduces the cross-sectional area of arterial lumen and changes the regional blood rheology thereby, which, in turn, severely affects human health. Atherosclerotic lesions are usually formed and grow at certain locations of the arterial tree, such as at curving, branching, and bifurcation sites. Although the exact causes for the initiation of a stenosis are not clearly known, it has been found that the proliferation of the disease is closely related to local hemodynamic factors, such as wall shear stress distribution (Smedby,<sup>1</sup> Liepsch<sup>2</sup>).

Over the past few decades, blood flow through stenotic arteries has drawn significant attention to the researchers and become an interesting research area to work on. Several researchers modeled steady flow of blood through stenosed rigid artery by considering the flowing blood as a Newtonian fluid (Lee,<sup>3</sup> Deshpande et al<sup>4</sup>). Because of the periodic pumping action of the heart, blood flow in arteries is pulsatile in nature. Treating blood as Newtonian fluid, Mustapha et al,<sup>5</sup> Liu et al,<sup>6</sup> Paul and Molla,<sup>7</sup> Mandal et al<sup>8</sup> investigated unsteady flow of blood through stenotic rigid artery with pulsation. Goswami et al<sup>9</sup> compared the simple and physiological pulsatile flow behaviors of blood in a stenosed artery. Considering blood as a non-Newtonian fluid, Mandal et al<sup>10</sup> investigated the pulsatile flow in a stenosed rigid artery.



# Numerical Simulation of Mass Transfer in Pulsatile Flow of Blood Characterized by Carreau Model under Stenotic Condition

Su. Mukhopadhyay<sup>1</sup>, M. Shankar Mandal<sup>2</sup> and Sw. Mukhopadhyay<sup>3</sup>

<sup>1</sup>Department of Mathematics, Acharya P. C. Roy Govt. College, Siliguri, W.B., India

<sup>2</sup>Department of Mathematics, G.G.D.C at Kalna-I, Purba Bardhaman-713405, W.B., India

<sup>3</sup>Department of Mathematics, The University of Burdwan, Burdwan-713104, W.B., India

†Corresponding Author Email: [manimath@yahoo.com](mailto:manimath@yahoo.com)

(Received June 3, 2020; accepted November 4, 2020)

## ABSTRACT

The present numerical study deals with a mathematical model representing mass transfer in blood flow under stenotic condition. Streaming blood is considered as a non-Newtonian fluid characterized by Carreau fluid model and the vessel wall is taken to be flexible. The nonlinear pulsatile flow phenomenon is governed by the Navier-Stokes equations together with the continuity equation while that of mass transfer is governed by the convection-diffusion equation coupled with the velocity field. A finite difference scheme is developed to solve these equations accompanied by suitable initial and boundary conditions. Results obtained are examined for numerical stability up to wanted degree of correctness. Various significant hemodynamic parameters are examined for additional qualitative insight of the flow-field and concentration-field over the entire arterial segment with the help of the obtained numerical results. Comparisons are made with the available results in open literature and good agreement has been achieved between these two results. Comparisons have been made to understand the effects of viscosity models for Newtonian and non-Newtonian fluids and also for rigid and flexible arteries.

**Keywords:** Non-Newtonian fluid; Carreau fluid model; Pulsatile flow; Mass transfer; Flexible artery.

## 1. INTRODUCTION

Partial occlusion of arteries, known as arterial stenosis, is one of the most frequent anomalies in cardiovascular system. Due to accumulation of low-density lipoprotein and other lipid bearing materials in streaming blood, such type of constrictions are formed (Ross 1993) and the disease thus caused by is called atherosclerosis. Under physiological conditions, atherosclerotic plaques may burst with no notice and as a result heart attack and stroke occur (Haque *et al.* 2014). Though the accurate grounds behind the commencement of such constriction are not yet clearly known but it is well recognized that once such constriction is shaped, the hemodynamic environment in the area of the constriction is drastically changed and fluid dynamic factors take part in the propagation of the disease (Friedman *et al.* 1992; Smedby 1997; Liepsch 2002). Such obstruction in arteries implies that the transport of low-density lipoproteins from blood stream onto the arterial wall must play a key role in the development of stenotic lesions. Moreover, mechanical stresses

are created by the interactions of plaque with the flow of blood leading to its burst. Recirculation region is formed downstream the plaque (Haque *et al.* 2014).

The flow disturbances associated with a medium degree of stenosis can be detected through the use of non-invasive methods such as the Doppler ultrasound technique, but a method to detect a mild stenosis is still out of hand. The ability to describe the flow through constricted arteries may provide the possibility of diagnosing the disease in its earlier stages, even before the stenosis become clinically relevant, and is the basis for surgical intervention. Computational fluid dynamics provides a useful and non-invasive tool to study the hemodynamic factors, suspected to be associated with the propagation of atherosclerosis, through stenosed arteries (Pontrelli 2001).

During the past few decades, several studies on fluid dynamics through constricted arteries have been carried out to evaluate the flow pattern and the wall shear stress under steady and pulsatile flow



# Heat transfer in pulsatile blood flow obeying Cross viscosity model through an artery with aneurysm

Subrata Mukhopadhyay · Mani Shankar Mandal · Swati Mukhopadhyay

Received: 6 October 2020 / Accepted: 11 September 2021  
© The Author(s), under exclusive licence to Springer Nature B.V. 2021

**Abstract** A mathematical model has been developed to represent heat transfer in pulsatile blood flow through an artery having an aneurysm in its lumen occurring from different types of anomalous enlargement. Time-variant wall geometry has been considered and the streaming blood is taken as a non-Newtonian fluid obeying Cross viscosity model. With the help of stream function–vorticity method, a finite difference scheme is used to solve the governing equations along with the suitable initial and boundary conditions in order to find out the physiologically noteworthy parameters up to the required degree of precision. Particular importance has accordingly been paid in comparing the current numerical results with the existing ones, and an excellent conformity between these two has been attained. For additional qualitative insight into the flow and heat transfer, effects of severity of aneurysm and different hemodynamic parameters on axial velocity, wall shear stress, and heat transfer rate are presented through graphical representations and analyzed in detail.

**Keywords** Aneurysm · Blood flow · Cross viscosity model · Heat transfer · Stream function–vorticity method

## 1 Introduction

From the theoretical, experimental, and clinical perspectives, blood flow through a diseased artery is an attractive field of research. Unusual expansion in the arterial wall is supposed by many researchers to play a major role in the formation of a disease, a leading cause of mortality in the present world. Several mathematical models have been constructed to explain the capacity to understand the accessible treatment. Weakening of vessel wall in certain locations forms a blood-filled balloon-like dilatation known as an aneurysm. Normal flow of blood through arteries thus becomes perturbed and complicated. Rhythmic flow of blood inside an aneurysm worsens the structure, and rupture may happen ultimately. The most common sites of aneurysm are the arteries such as cerebral, carotid,

---

S. Mukhopadhyay  
Department of Mathematics, Acharya P. C. Roy Govt. College, Siliguri, W.B., India

M. S. Mandal (✉)  
Department of Mathematics, G.G.D.C at Kalna-I, Purba Bardhaman, W.B. 713405, India  
e-mail: manimath@yahoo.com

S. Mukhopadhyay  
Department of Mathematics, The University of Burdwan, Burdwan, W.B. 713104, India

Published online: 18 October 2021





# Pulsatile flow of blood with shear-dependent viscosity through a flexible stenosed artery in the presence of body acceleration

Subrata Mukhopadhyay<sup>1</sup> · Mani Shankar Mandal<sup>2</sup> · Swati Mukhopadhyay<sup>3</sup>

Received: 3 December 2020 / Revised: 20 November 2021 / Accepted: 17 January 2022  
© The Korean Society of Rheology 2022

## Abstract

A mathematical model of physiological pulsatile flow of blood through a stenotic flexible artery in the presence of body acceleration is presented in this paper. Streaming blood is considered as a shear-thinning non-Newtonian fluid as proposed by Yeleswarapu (Evaluation of continuum models for characterizing the constitutive behaviour of blood, Ph.D. thesis, Dept. Mech. Eng., University of Pittsburgh, 1996), and a physiological pulsatile flow rate proposed by Pedrizzetti (J Fluid Mech 310:89–111, 1996) has been taken through the tube. Deformation of vessel wall is modelled as a function of flow rate. This computational study of an idealized model may bring some insights for realistic blood flow through a stenotic artery. The novelty of this work lies in the fact that realistic flow of blood through a stenosed artery has been studied as far as possible and a new idea has been provided to describe the arterial wall motion. Governing equations in cylindrical polar coordinates are solved using stream function–vorticity method. Behaviour of various flow quantities is investigated through a parametric study. It is noted that the degree of constriction and body acceleration have important impacts on the haemodynamic parameters such as wall shear stress, oscillatory shear index, and relative residence time. Increasing body acceleration enhances the peak value of wall shear stress, but reduces the oscillatory shear index and relative residence time. Almost 1/4th increase in length of flow separation is found when Froude number raises its value from 0.1 to 0.5, other parametric values remaining fixed. On the other hand, almost 50% increase in the magnitude of the peak value of wall pressure is found when the amplitude of body acceleration takes a value 0.4 ( $A = 0.4$ ) compared to the without body acceleration case ( $A = 0$ ). These results have a significant role.

**Keywords** Non-Newtonian fluid · Pulsatile flow · Stenotic artery · Body acceleration · Finite difference

## 1 Introduction

Studies related to flow of blood through stenotic arteries have drawn noteworthy attention to researchers during the last few decades, as the commencement and progress of many cardiovascular diseases which direct to the malfunction of the cardiovascular system are intimately associated with the haemodynamics of such an artery. Atherosclerosis or arterial stenosis commences through alteration in

endothelial cell utility that encourages white blood cells to attach to the endothelium instead of flowing through the blood usually and the endothelium turns out to be damaged thereby. This permits blood cells and deadly substances, such as lipids, low density lipoproteins and triglycerides, present in the blood to go by the endothelium and mount up in this area. Different compound phenomena take place in the course of time and lastly calcium builds up over the wound site to shape a material similar to bone.

The formation of arterial stenosis reduces the supply of blood to the distal bed through that artery. With the spread of atherosclerotic plaque, if it becomes unstable, it ruptures and exposes its contents to streaming blood. Platelets may then build up around the ruptured plaque and result in blood coagulation which occludes the artery. When blood flowing through an artery is severely compromised by a blood clot, the cells of the tissues that depend on the blood flow from that artery become injured or die. Coronary atherosclerosis

✉ Mani Shankar Mandal  
manimath@yahoo.com

<sup>1</sup> Department of Mathematics, Acharya P. C. Roy Govt. College, Siliguri, WB, India

<sup>2</sup> Department of Mathematics, G.G.D.C at Kalna-I, Purba Bardhaman, WB 713405, India

<sup>3</sup> Department of Mathematics, The University of Burdwan, Burdwan, WB 713104, India



# The flow of MHD Maxwell liquid over an extending surface with variable free stream temperature

Mani Shankar Mandal<sup>1</sup>  | Swati Mukhopadhyay<sup>2</sup>

<sup>1</sup>Department of Mathematics, Govt. General Degree College, Purba Bardhaman, India

<sup>2</sup>Department of Mathematics, The University of Burdwan, Burdwan, India

## Correspondence

Mani Shankar Mandal, Department of Mathematics, Govt. General Degree College, Kalna-I, Purba Bardhaman 713405, India.  
Email: manimath@yahoo.com

## Abstract

The aim of this article is to investigate the steady two-dimensional flow of a magnetohydrodynamics Maxwell liquid over a stretched exterior having prescribed exterior hotness in the presence of changeable free stream temperature. Here the Maxwell liquid replica reflects non-Newtonian liquid behavior. By means of similarity alterations, the leading differential equations which are partial in nature are altered to differential equations that are ordinary in nature and subsequently numerical solutions are received via a shooting scheme. The motion and heat transport characteristics for the leading parameters is scrutinized and talked about elaborately using their graphical demonstration. It is established that the liquid speed reduces with escalating magnetic parameter. The consequences of growing Maxwell parameter are to hold back the speed of liquid. An increment in heat movement speed has been observed for enhancing the temperature ratio and Maxwell parameters.

## KEYWORDS

boundary layer, Maxwell fluid, MHD, non-Newtonian fluid, shooting method, stretching surface, variable free stream temperature



# Cu–water nanofluid flow with arbitrarily shaped nanoparticles over a porous plate in a porous medium in the presence of slip

SWATI MUKHOPADHYAY<sup>1</sup> \*, MANI SHANKAR MANDAL<sup>2</sup> and K VAJRAVELU<sup>3</sup>

<sup>1</sup>Department of Mathematics, The University of Burdwan, Burdwan 713 104, India

<sup>2</sup>Department of Mathematics, Govt. General Degree College, Kalna-I, Burdwan 713 405, India

<sup>3</sup>Department of Mathematics, University of Central Florida, Orlando, FL 32816-1364, USA

\*Corresponding author. E-mail: swati\_bumath@yahoo.co.in

MS received 25 February 2022; revised 30 May 2022; accepted 10 June 2022

**Abstract.** The objective of the article is to analyse the forced convection nanofluid flow over a permeable plate in an absorbent medium using slip boundary conditions. A single-phase model for the nanofluid is used with variable shapes of nanoparticles. The partial differential equations (PDEs) of the model are altered into a set of non-linear ordinary differential equations (ODEs) by a suitable alteration. To obtain the solutions of the system of equations numerically, Runge–Kutta method is used with a shooting technique. The effects of various parameters, like permeability, suction/injection, nanoparticle volume fraction, velocity slip, thermal slip and nanoparticle shape parameters on velocity and temperature profiles are presented graphically and analysed. In addition, for a clear understanding of the model, the flow and the heat transfer characteristics are presented through graphs and analysed. Fluid velocity is found to increase with the increasing values of permeability of the porous medium, whereas temperature is found to reduce in this case. Temperature is a rising function of the thermal slip parameter, whereas it is a decreasing function of the velocity slip parameter.

**Keywords.** Nanofluid; forced convection; porous medium; arbitrarily shaped nanoparticles; partial slips; permeable plate.

**PACS Nos** 44.20.+b; 44.27.+g; 44.30.+v

## 1. Introduction

Nanotechnology has attracted enormous attention among researchers due to its widespread applications in manufacturing processes and medical sciences. Nanofluid is an innovative class of fluids, the concept for which has been planned as a foundation for increased performance of heat transport fluids. To augment the thermal conductivity of the base fluid (viz., water, ethylene glycol, oil), which has low thermal conductivity, nanometre-sized elements are scattered in the base liquid. To permit more heat transfer, usually, these elements are prepared of metal or metal oxide to supplement the conduction and convection coefficients. Choi [1] was the first to understand that the accumulation of nanoparticles in the base liquid tremendously augments the thermal conductivity of the liquid. Buongiorno [2] first studied the convective heat transfer in nanofluids by observing the augmentation in thermal conductivity owing to Brownian movement and thermophoretic dispersion of

nanoparticles. Later, Tiwari and Das [3] initiated a novel replica that is well accepted nowadays. Mustafa *et al* [4], Nadeem *et al* [5,6], Hussain *et al* [7], Das *et al* [8], Mabood *et al* [9], Hayat *et al* [10] and many others have investigated various aspects of nanofluid flow and heat transfer.

All these researchers considered no-slip conditions at the boundary. However, the no-slip condition at the boundary is unsuitable in situations where the exterior is sufficiently glossy. Partial slip speed is usually present at the border for emulsions, suspensions, foams, polymer solutions etc. Slip velocity at the boundary also occurs in a variety of situations, particularly in perforated plates and nets finished by wires, greased or chemically indulged hydrophobic surfaces, rough or porous surfaces (Hafidzuddin *et al* [11]), and amazing hydrophobic nanosurfaces (Choi and Kim [12], Hafidzuddin *et al* [11]). Applications of slip at the boundary are used in simulated heart valves, compound fluid problems and fluid flow on many interfaces. Hayat *et al* [13] managed



# Some aspects of flow over a non-isothermal unsteady stretched exterior fixed in porous medium among heat production/amalgamation

Mani Shankar Mandal<sup>a,\*</sup>, Swati Mukhopadhyay<sup>b</sup>

<sup>a</sup> Department of Mathematics, Govt. General Degree College, Kalna-I, Purba Bardhaman, 713405, India

<sup>b</sup> Department of Mathematics, The University of Burdwan, Burdwan, 713104, West Bengal, India

## ARTICLE INFO

### Keywords:

Boundary layer flow  
Unsteady stretched surface  
Variable temperature  
Porous medium  
Heat generation/absorption

## ABSTRACT

The aspire of this article is to examine the consequences of warmth production/amalgamation on border level stream of sticky liquid due to an uneven stretched piece in absorbent medium having a variable exterior hotness. Resemblance answers of the altered leading equations are attained and the numerical answers of those self-alike equations are gained by shooting process. The characteristics of stream and warmth transport for dissimilar data of related parameters are examined and analyzed in detail. Liquid speed and hotness reduce due to rising values of unsteadiness parameter. Liquid speed reduces by means of the rising permeability parameter of the medium ensuing an augmentation in thermal meadow in stable and uneven cases. Flow field is unaltered by the indexes that cause the temperature disparity but it is worth mentioning that the indexes which settle on the disparity of the piece hotness respectively with space and instance, both be liable to boost the hotness disparity and in that way also the Nusselt number. Speed of heat transport augments with growing heat foundation parameter. Thus uneven stretched sheet makes the replica appropriate to a range of manufacturing sectors.

## Nomenclature

$k(t)$	time dependent permeability
$k_0$	early permeability
$T$	Time
$c$	optimistic constant
$U(x, t)$	non-uniform speed
$u$ and $v$	velocity components along $x$ and $y$ directions respectively
$T$	hotness of the liquid
$\kappa$	thermal diffusivity
$Q_0$	original value of heat creation/amalgamation coefficient
$Q(t) > 0$	heat generation
$Q(t) < 0$	heat absorption
$c_p$	specific heat of the fluid
$T_0$	initial temperature
$T_w$	exterior temperature
$T_\infty$	steady ambient hotness
$d$	proportionality constant
$r, s$	Power law indices
$M$	unsteadiness parameter
$k_1$	permeability of the medium
$Pr$	Prandtl number
$Nu_x Re_x^{-\frac{1}{2}}$	Local Nusselt number
$f'(\eta)$	Fluid speed

(continued on next column)

(continued)

$f''(0)$	skin-friction coefficient
Greek Symbol	
$\alpha$	optimistic constant
$\mu$	viscosity of the liquid
$\rho$	density of the liquid
$\nu$	kinematic viscosity
$\psi$	stream function
$\theta$	dimensionless temperature
$\eta$	Similarity variable
$\lambda$	heat source/sink parameter
$\theta(\eta)$	Hotness

## 1. Introduction

In the previous decades, the border level stream of various kinds of fluids trapped the curiosity of a number of researchers due to its significance from technological opinion. The transfer of warmth, momentum, and accumulation is of immense interest.

The studies connected to the problems of extended sheets have numerous applications in modern industries. Applications of elongated surfaces are found in the regions of paper production, continuous casting of metals, hot rolling, glass blowing and cable sketching [1–3]. Instances

\* Corresponding author.

E-mail address: [manimath@yahoo.com](mailto:manimath@yahoo.com) (M.S. Mandal).



# Influence of Thompson and Troian slip on the nanofluid flow past a permeable plate in porous medium

SUDIP DEY<sup>1</sup>, SWATI MUKHOPADHYAY<sup>1</sup> \* and MANI SHANKAR MANDAL<sup>2</sup>

<sup>1</sup>Department of Mathematics, The University of Burdwan, Burdwan 713 104, India

<sup>2</sup>Department of Mathematics, Govt. General Degree College, Kalna-I, Burdwan 713 405, India

\*Corresponding author. E-mail: swati\_bumath@yahoo.co.in

MS received 28 April 2022; revised 24 November 2022; accepted 5 December 2022

**Abstract.** The aim of the present analysis is to study the influence of Thompson and Troian slip on forced convective nanofluid flow over a permeable plate in Darcy porous medium in the presence of zero nanoparticle flux at the boundary. By the appropriate make-over, the foremost partial differential equations (PDEs) are abridged to ordinary differential equations (ODEs) and numerical solutions for the nonlinear equations are subsequently attained by shooting technique. Due to enhanced permeability parameter, speed and concentration of the liquid increase but the width of the momentum boundary layer and temperature reduce. The current analysis discloses that by reducing the width of the boundary level, the rising (velocity) slip parameter forces the fluid speed and concentration to increase while dimensionless temperature reduces for increasing (velocity) slip. Compared to blowing, liquid speed and concentration are superior for suction. With the rise in Brownian motion parameter, concentration diminishes whereas with the rise in thermophoresis parameter, temperature is found to rise. The results achieved in this examination expose various motivating characteristics which demand additional investigation of the problem.

**Keywords.** Forced convection; nanofluid; Darcy porous medium; Thompson–Troian slip; velocity slip.

**PACS Nos** 44.27.+g; 47.56.+r; 44.20.+b

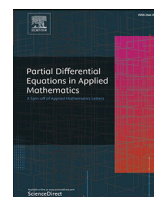
## 1. Introduction

Nanofluid is an exclusive type of fluid with nanosized elements suspended in a foundation fluid, viz., water and ethylene glycol having very poor thermal conductivity [1]. The concept of nanofluid was first introduced by Choi [2]. Basically, it is a liquid suspension containing ultrafine elements (normally metal or metal oxide) of diameter lesser than 50 nm. Because of the presence of these nanosized particles, heat conduction of the suspension is highly enhanced. So, these nanofluids are used in many manufacturing and engineering procedures and have a large range of applications, viz., chemical engineering, biomedical, geophysical science, electronic cooling, etc. [1–5]. A number of applications of the nanofluids can be found in [6–12].

Heat and mass transportation inside the porous media play a significant role in miscellaneous fields, viz. in petroleum engineering, geothermal processes and several other fields. Convection throughout the absorbent medium can be beneficial to many areas, such as

filling design, granule storeroom, geothermal arrangements, high-temperature traders, strain strategy, metal dispensation etc. Significant attention has been paid to investigate nanofluid flow, heat and mass transport properties for flow past a plate in a permeable medium [13]. Extensive reviews on convection through porous media can be found in Waini *et al* [14], Singha *et al* [1], Sheikholeslami *et al* [15], Elgazery *et al* [16], Sreedevi *et al* [17] etc. For small porosity, Darcy law holds good.

The no-slip boundary conditions are generally assumed in most of the articles found in open literature. But the existence of no-slip at the solid–fluid interface is a subject of huge debate [18]. The non-adherence of liquid towards a hard border, recognised as the (velocity) slip, is generally experienced below some fixed conditions [19]. Slip flow occurs in microscale system and it differs from the usual flow [20]. Though slip flow has wider applications, but only a few studies are available on slip flow. The heat transport in a stream past a plate with slip condition at the boundary was examined by Martin and Boyd [21]. Ellahi *et al* [22] obtained



# MHD mixed convective Maxwell liquid flow passing an unsteady stretched sheet

Iswar Chandra Mandal<sup>a</sup>, Swati Mukhopadhyay<sup>a\*</sup>, Mani Shankar Mandal<sup>b</sup>

<sup>a</sup>Department of Mathematics, The University of Burdwan, Burdwan, West Bengal 713104, India

<sup>b</sup>Department of Mathematics, Govt. General Degree College, Kalna-I, Purba Bardhaman 713405, India

## ARTICLE INFO

### Keywords:

Maxwell fluid  
Unsteady flow  
MHD  
Mixed convection  
Stretching sheet

## ABSTRACT

This study is aimed to investigate the unsteady two-dimensional combined forced and natural convective motion of MHD Maxwell liquid passing a stretched surface. Non-Newtonian feature of the liquid is portrayed with the 'upper convected Maxwell fluid' model. A prearranged surface 'temperature' at the boundary is considered. 'Similarity transformations' are applied to find the self-similar solutions and then the altered 'equations' are numerically solved with the help of 'Runge-Kutta method with shooting technique'. The behaviours of 'flow' and 'heat' transportation characters for diverse foremost 'parameters' are examined and discussed in detail for clear understanding about the non-Newtonian flow features and related thermal field. The interesting behaviours of flow and heat transfer characters warrant further studies of this problem for other type of situations. Due to unsteadiness, 'velocity and temperature' are seen to reduce for 'buoyancy aided and opposed' flows. Also, it is noted that for 'buoyancy aided and opposed flows', liquid velocity diminishes for rising 'Hartmann number'. The influences of mounting 'Maxwell parameter' are to delay fluid velocity for either value of mixed convection parameter. However, for this case temperature is improved. Owing to mixed convection, liquid velocity increases but the temperature reduces within the 'boundary layer' area. Here lies the significance of the present study as this result is very much helpful for industrial applications for cooling purpose of different devices.

## 1. Introduction

'Mixed convection' (collective 'forced and natural convections') motions occur in a lot of transportation procedures in environment and in manufacturing policies.<sup>1</sup> 'Mixed convection' is provoked by the movement of a hard substance (for 'forced convection') and 'buoyancy' (for 'natural convection'). It is eminent that the 'buoyancy force' branching from warming or chilling of the uninterrupted stretched piece alter the stream and the temperature of fluid and thus the 'heat' transport characters of industrialized procedures. Owing to the modern progress in nuclear energy, space technology, electronics investigations on 'buoyancy' provoked motions of 'non-Newtonian liquid' goes on to be a key region of attention. A number of significant sensible uses engage the guess of ecological contamination in the sea and environment, 'thermal stratification' in ponds, in addition to growth and plan of compound dispensation apparatus.<sup>2</sup> The problems of mutual 'forced and free' convections in a 'boundary layer' passing an uninterrupted 'moving' exteriors can be found in Refs.<sup>3–8</sup>

All of the aforesaid investigations were limited to flow of 'Newtonian

liquids'. In contrast, 'non-Newtonian liquids' have turned into progressively significant technologically in recent years (Rajagopal et al.<sup>9</sup>). The hypothesis of 'non-Newtonian liquids' has turn out to be a meadow of extremely lively research for the previous decades since this type of liquids stands for, precisely, a lot of technologically significant liquids viz. mock threads in engineering and artificial films. Although, substantial development has been completed in accepting the flow occurrence, additional research are desirable to recognize the consequences of a variety of 'parameters' occupied in the 'non-Newtonian fluid' replicas.<sup>10</sup> As of the complication of these liquids, not a sole 'constitutive equation' exhibiting every feature of such type of liquids. In text, the greater part of 'non-Newtonian fluid' replicas are based on straightforward replicas viz. 'power-law' model and 'grade two or three'.<sup>11–13</sup> 'Power-law model' has huge applications in modelling liquids having 'shear-dependent viscosity' however it is not capable to forecast the influences of flexibility. The liquids of 'grade two or three' can compute the influences of flexibility however the 'viscosity' in these replicas is shear independent, and also they are not successful to forecast the influences of 'stress relaxation'. A subtype of 'rate type fluids', like

\* Corresponding author.

E-mail address: [swati\\_bumath@yahoo.co.in](mailto:swati_bumath@yahoo.co.in) (S. Mukhopadhyay).

<https://doi.org/10.1016/j.padiff.2024.100644>

Received 19 December 2023; Received in revised form 22 January 2024; Accepted 8 February 2024

Available online 9 February 2024

2666-8181/© 2024 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

## EFFECTS OF MAGNETIC FIELD ON HEAT AND MASS TRANSFER IN PULSATILE FLOW OF BLOOD UNDER STENOTIC CONDITION

SUBRATA MUKHOPADHYAY<sup>1</sup>, MANI SHANKAR MANDAL<sup>2\*</sup>,  
SWATI MUKHOPADHYAY<sup>3</sup>

<sup>1</sup>*Department of Mathematics, Acharya P.C. Roy Govt. College,  
Siliguri, W.B., India*

<sup>2</sup>*Department of Mathematics, G.G.D.C at Kalna-I,  
Purba Bardhaman-713405, W.B., India*

<sup>3</sup>*Department of Mathematics, The University of Burdwan,  
Burdwan-713104, W.B., India*

[Received: 8 May 2021]

**ABSTRACT:** A mathematical model is developed in the present study to investigate the heat and mass transfer phenomena in blood flow under stenotic condition. The non-Newtonian Carreau fluid model is used to characterize the streaming blood. The nonlinear governing equations are solved numerically by employing a finite difference scheme along with suitable initial and boundary conditions under the action of applied magnetic field. Various significant hemodynamic parameters are examined for additional qualitative insight of the flow-field, temperature-field and concentration-field over the entire flow regime with the help of the numerical results obtained in this study. Comparisons are made with available results in open literature and are found in good agreement between these two results.

**KEY WORDS:** Heat and mass transfer, Non-Newtonian fluid, Magnetic field, Finite-difference.

### 1 INTRODUCTION

Heat and mass transfer phenomena in constricted channels have significant practical implications in engineering fields and Biorheology. The motion of an electrically conducting fluid across a magnetic field induces current, which affects the fluid flow. In fact the propagating field is influenced by Lorentz force. Understanding the physics of magnetohydrodynamic (MHD) flow, prediction of flow separation region

---

\*Corresponding author e-mail: [manimath@yahoo.com](mailto:manimath@yahoo.com)