An application of similarity Measure of soft sets in decision Making

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Outlines

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Abstract

Soft set, as a parametrized family of subsets of a crisp universal set, has more ability to handle uncertainty condition. Similarity between two sets helps us to compare the nature of two sets. In this paper , I give an application of similarity measure of soft sets to handle for a decision-making problem in a real-life situation.

Introduction

There are several techniques to represent and solve various types of uncertainties prevailing in this physical world. In 1999 Molodstov[6] has introduced the concept of soft sets. The soft set theory has a great potential for solving so many real-life problems.

Again, in several problems, it is often needed to compare two sets. Majumdar & Samanta[2] has given some techniques of similarity measure of soft sets. In this paper I am going to present an application in decision making using similarity measurement of two soft sets.

Preliminaries

Definition[6]: Let U be an initial universe and E be a set of parameters. Let $\wp(U)$ denotes the power set of U and $A \subset E$. A pair (F, A) is called a soft set if and only if F is a mapping of A into $\wp(U)$.

Example: Suppose U is the set of house under consideration. E is, a phrase or a sentence. Suppose that there are six houses in the universal set, given by $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters,

where e_1 =expensive, e_2 =beautiful, e_3 =chief, e_4 =in green surroundings.

Suppose that $F: E \to \wp(U)$ is given by $F(e_1) = \{h_1, h_2\}$, $F(e_2) = \{h_1, h_2, h_3\}$, $F(e_3) = \{h_3, h_6\}$, $F(e_4) = \{h_1, h_4, h_6\}$. Here (F, E)describes a soft set such that the houses h_1, h_2 are expensive; the houses h_2, h_3, h_4 are $beautiful$; the houses h_3, h_6 are cheap and the houses h_1, h_4, h_6 are in green $surr$oundings.

Matrix representation of soft set[7]

Let (F, E) be a soft set over the universe U, where $U = \{x_1, x_2, \dots, x_m\}$ and $E = \{e_1, e_2, \dots, e_n\}$. Then the soft set (F, E) can be represented by a $m \times n$ matrix such that the $(i, j)^{th}$ entry of the matrix is 1 if $x_i \in F(e_j)$ and it is 0 if $x_i \notin F(e_j)$.

Example: Let us consider the soft set (F, E) over the universe, where $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3, e_4\}$ such that $F(e_1) = \{x_1, x_2\}, F(e_2) = \{x_3\}, F(e_3) = \{x_1, x_3\}, F(e_4) = \{x_2\}$.
Then the matrix representation of this soft set is given by

(F, E)	e_1	e_2	e_3	e_4
x_1	1	0	1	0
x_2	1	0	0	1
x_3	0	1	1	0

Here we denote j^{th} column of the matrix by the vector $\vec{F}(e_j)$, e.g. here $\vec{F}(e_1) = (1,1,0)$.

Similarity measure of soft sets[9]

Let (F, A) and (G, B) be two soft sets over a common universe U. If $A = B$, then similarity between (F, A) and (G, B) is defined by

$$S(F,G) = \frac{\sum_i \vec{F}(e_i) \cdot \vec{G}(e_i)}{\sum_i [\vec{F}(e_i)^2 \vee \vec{G}(e_i)^2]}$$

If $A \neq B$ and $E = A \cup B \neq \phi$, then we first define $\vec{F}(e) = \underline{0}$ for $e \in E/A$ and $\vec{G}(e) = \underline{0}$ for $e \in E/B$. Then $S(F,G)$ is defined by the above formula.

Example: Let us consider two soft sets (F, E) and (G, E) over U, where $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$. The matrix representation of these two soft sets are given by

$$(F, E) = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } (G, E) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Then the similarity measurement of these two soft sets are given by $S(F, G) = \frac{\sum_{i=1}^{3} \vec{F}(e_i)\cdot\vec{G}(e_i)}{\sum_{i=1}^{3} [\vec{F}(e_i)^2 \vee \vec{G}(e_i)^2]} = \frac{1}{3}$

Note: If $S(F, G) \geq 0.5$, then the two soft sets are called significantly similar. So in the above case the two soft sets are not significantly similar.

Theorem[2] : Let (F, A) and (G, B) be two soft sets over the same finite universe U. Then the followings hold:

i) $S(F, G) = S(G, F)$

ii) $0 \leq S(F, G) \leq 1$

iii) $S(F, F) = 1$

The proof is trivial.

An application in decision making

Suppose Sourav wants to buy a house at Burdwan. He goes to a real estate company. The company has to find suitable house for Sourav based on some criterions.

Suppose the company has two houses, H_1 and H_2. Now the problem is which house is suitable for Sourav.

Now the company creates a set $U = \{1,2\}$, where 1 indicates $'Strong\ preference'$ or $'Yes'$ and 2 indicates $'Less\ preference'$ or $'No'$.

Also the company considers a parameter set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

Where

e_1 = near rail station

e_2 = in green surroundings

e_3 = well furnished

e_4 = low cost

e_5 = car parking available

e_6 = place for gardening available

e_7 = near hospital

$e_8 = south\ facing$

Based on Sourav's preferences the company creates a soft set which is

$Sourav$	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
1	0	1	0	1	0	1	0	1
2	1	0	1	0	1	0	1	0

The soft set indicates that Sourav strongly preference that the house should be *'In green surroundings'*, *'Low cost'*, *'Place for gardening available'*, *'South facing'*.

Again based on the condition of houses H_1 and H_2 the company creates two soft sets for house H_1 and house H_2 . The soft set for H_1 is

H_1	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
1	1	0	0	0	1	0	0	0
2	0	1	1	1	0	1	1	1

i.e. the houses H_1 is '$near\ rail\ station$',
'$carparking\ available$' and is not '$in\ green\ surroundings$', is not '$well\ furnished$', is not '$low\ cost$', no '$place\ for\ gardening\ available$', is not '$near\ hospital$'.

Here similarity measure between the soft sets of Sourav and H_1 is given by

$$S(Sourav, H_1) = 0.25 < 0.5$$

So the two soft sets are not significantly similar.

Also for the set for H_2 is

H_2	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
1	0	1	1	1	1	1	0	1
2	1	0	0	0	0	0	1	0

i.e. the house H_2 is '$in\ green\ surroundings$' is '$well\ furnished$', is of '$low\ cost$', has '$car\ parking\ available$', has '$place\ for\ gardening$', is '$south\ facing$' and the house H_2 is not '$near\ hospital$'.

Here the similarity measure between the soft sets of Sourav and H_2 is given by $S(Sourav, H_2) = 0.75 > 0.5$.

So the two soft sets are significantly similar.

So probably the company choose the house H_2 for Sourav.

Conclusion

In recent some researchers have given the concept of type 2 soft set which is more useful to deal with problem of uncertainties. Also there are various scope of research like similarity, matrix representation, relations of type 2 soft sets and their applications.

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